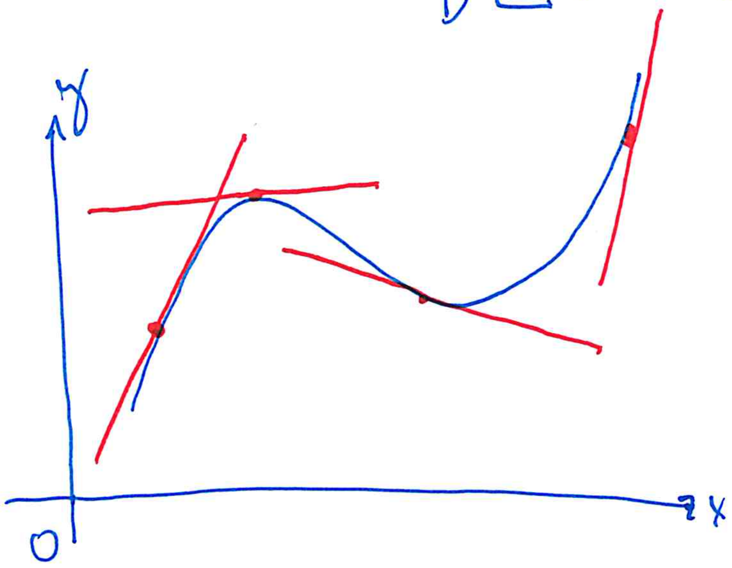
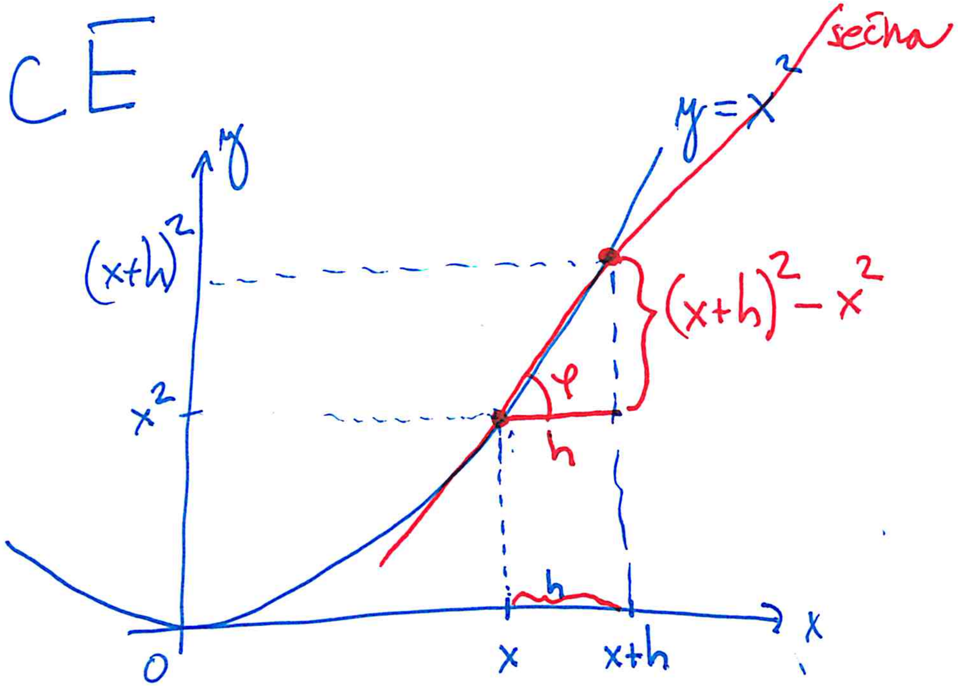
DERIVACE





$$k_S = \mathrm{tg}\,\varphi = \frac{(x+h)^2 - x^2}{h} = \frac{\cancel{x^2} + 2\cancel{h}x + h^{\cancel{2}} - \cancel{x^2}}{\cancel{h}} = 2x + h$$

směrnice tečny $k_T = \lim_{h \to 0} k_S = \lim_{h \to 0} (2x+h) = \underline{2x}$

Derivace funkce $f(x)$ v bodě a je směrnice tečny sestrojené v bodě a ke grafu funkce $f(x)$:

$$f'(a) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$f(x)$	$f'(x)$
c	0
x	1
x^2	$2x$
x^n	$n.x^{n-1}$
e^x	e^x
$\ln x$	$\frac{1}{x}$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\operatorname{arctg} x$	$\frac{1}{1+x^2}$

$(x^4)' = 4x^3 \qquad (\sqrt{x})' = (x^{\frac{1}{2}})' = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}},\ x>0$

Mají-li funkce f a g v bodě a derivaci, mají v bodě a derivaci i funkce $f+g$, $f-g$, $f\cdot g$ a platí

$$(\alpha\cdot f)'(a) = \alpha\cdot f'(a)$$

$$(f+g)'(a) = f'(a) + g'(a)$$

$$(f-g)'(a) = f'(a) - g'(a)$$

$$(f\cdot g)'(a) = f'(a)\cdot g(a) + f(a)\cdot g'(a)$$

Je-li navíc $g(a) \neq 0$, má derivaci i $\frac{f}{g}$ a platí

$$\left(\frac{f}{g}\right)'(a) = \frac{f'(a)\cdot g(a) - f(a)\cdot g'(a)}{g^2(a)}$$

$$(2x^2+3x-7)' = (2x^2)' + (3x)' - (7)' = 2(x^2)' + 3(x)' - (7)' = 4x+3$$

$$(x^4-5x^3)' = 4x^3 - 5\cdot 3x^2 = 4x^3 - 15x^2$$

$$(3\sqrt[2]{x} - 5\sqrt[5]{x})' = 3\left(x^{\frac{1}{2}}\right)' - 5\left(x^{\frac{1}{5}}\right)' = 3\,\frac{1}{2}\,x^{-\frac{1}{2}} - 5\,\frac{1}{5}\,x^{-\frac{4}{5}} = \frac{3}{2\sqrt{x}} - \frac{1}{\sqrt[5]{x^4}}\;,\; x>0$$

$$(3\sin x + 5\cos x)' = 3\cos x - 5\sin x$$

$$(x\sin x)' = (x)'\sin x + x(\sin x)' = \sin x + x\cos x$$

$$(e^x \ln x)' = e^x \ln x + e^x\,\frac{1}{x}\;,\; x>0$$

$$\left(\frac{x^2+1}{x+1}\right)' = \frac{(x^2+1)'(x+1) - (x^2+1)(x+1)'}{(x+1)^2} = \frac{2x(x+1) - (x^2+1)}{(x+1)^2}$$

Derivace složené funkce

Má-li funkce g v bodě a derivaci $g'(a)$ a má-li h v bodě $g(a)$ derivaci $h'(g(a))$ a je-li $f = h(g)$, má f v bodě a derivaci a platí

$$\boxed{f'(a) = (h(g))'(a) = h'(g(a)) \cdot g'(a)}$$

Příklady:

$$(\sin(x^2+1))' = \cos(x^2+1) \cdot 2x$$

$$\left(e^{x^3}\right)' = \left(e^{(x^3)}\right)' = e^{x^3} \cdot 3x^2$$

$$(\ln(3x-1))' = \frac{1}{3x-1} \cdot 3 \quad (3x-1>0)$$

$$\left[\cos\left(e^{1-x^3}\right)\right]' = -\sin\left(e^{1-x^3}\right) \cdot \left(e^{1-x^3}\right)' = -\sin\left(e^{1-x^3}\right) \cdot e^{1-x^3} \cdot (-3x^2)$$