Integrování substituční metodou

Jestliže $g' = g'(x)$ je spojitá funkce na $\langle a, b \rangle$ a $f = f(z)$ je spojitá pro každé $z = g(x)$, $x \in \langle a, b \rangle$ platí

$$\int f(g(x)) \cdot g'(x)\, dx = \int f(z)\, dz$$

prakticky: substituce

$$z = g(x)$$
$$dz = g'(x)\, dx$$

$$\int \boxed{2x} \sin(x^2) \boxed{dx} = \int \sin z \, dz = -\cos z + c = \underline{\underline{-\cos x^2 + c}}$$

$z = x^2$

$dz = 2x\,dx$

$$\int x^2 \cdot \sqrt{x^3+1}\,dx = \frac{1}{3}\int \sqrt{t}\,dt = \frac{1}{3}\int t^{\frac{1}{2}}\,dt = \frac{1}{3}\,\frac{t^{\frac{3}{2}}}{\frac{3}{2}} + c =$$

$$= \frac{1}{3}\cdot\frac{2}{3}(x^3+1)^{\frac{3}{2}} + c = \underline{\frac{2}{9}\sqrt{(x^3+1)^3} + c}$$

$t = x^3 + 1$

$dt = 3x^2\,dx \quad /:3$

$\frac{1}{3}dt = x^2\,dx$

$$\int \frac{2x}{\underbrace{x^2+1}} dx = \int \frac{1}{t} dt = \ln t + c = \ln(x^2+1) + c \quad , x \in \mathbb{R}$$

$$t = \underbrace{x^2+1}$$

$$dt = 2x\,dx$$

$$\int tg\,x\,dx = \int \frac{\sin x}{\boxed{\cos x}} dx = -\int \frac{1}{r} dr = -\ln|r| + c = -\ln|\cos x| + c$$

$$r = \boxed{\cos x}$$

$$dr = -\sin x\,dx \quad / \cdot (-1)$$

$$-dr = \sin x\,dx$$

$$\int \cos 3x\,dx = \frac{1}{3}\sin 3x + c \qquad (\sin 3x)' = 3\cos 3x$$

$$\int \cos 3x\,dx = \frac{1}{3}\int \cos t\,dt = \frac{1}{3}\sin t + c = \frac{1}{\boxed{3}}\sin 3x + c$$

$t = 3x$

$dt = 3\,dx \quad /:3$

$\frac{1}{3}dt = dx$

$$\int e^{5x-1}\,dx = \frac{1}{5}\int e^{r}\,dr = \frac{1}{5}e^{r} + c = \frac{1}{\boxed{5}}e^{5x-1} + c$$

$r = 5x - 1$

$dr = 5\,dx \quad /:5$

$\frac{1}{5}dr = dx$

$$\int f(x)\,dx = F(x) + c \rightarrow \int f(ax+b)\,dx = \frac{1}{a}F(ax+b) + c$$

$$\int e^{7x+5}\,dx = \frac{1}{7}e^{7x+5} + c$$

$$\int \sin(2-7x)\,dx = -\cos(2-7x)\cdot\left(-\frac{1}{7}\right) + c$$

$$\int \sqrt{3x-5}\,dx = \int (3x-5)^{\frac{1}{2}}\,dx = \frac{(3x-5)^{\frac{3}{2}}}{\frac{3}{2}}\cdot\frac{1}{3} + c$$

$$\int \frac{1}{2-1x}\,dx = \ln|2-x|\cdot\left(\frac{1}{-1}\right) + c$$

$$\int x^2 \cdot e^{3x}\,dx = \frac{1}{3}x^2 e^{3x} - \frac{2}{3}\int x e^{3x}\,dx =$$

$u = x^2 \quad u' = 2x$

$v' = e^{3x} \quad v = \underline{e^{3x} \cdot \frac{1}{3}}$

$u = x \quad u' = 1$

$v' = e^{3x} \quad v = \underline{e^{3x} \cdot \frac{1}{3}}$

$$= \frac{1}{3}x^2 e^{3x} - \frac{2}{3}\Big(\frac{1}{3}x e^{3x} - \frac{1}{3}\underbrace{\int e^{3x}\,dx}_{= \frac{1}{3}e^{3x}}\Big) = \frac{1}{3}x^2 e^{3x} - \frac{2}{9}x e^{3x} + \frac{2}{27}e^{3x} + c$$

$$\int \sin^4 x \cdot \cos x\,dx = \int t^4\,dt = \frac{t^5}{5} + c = \frac{\sin^5 x}{5} + c$$

$t = \sin x$

$dt = \cos x\,dx$

$\sin^5 x = (\sin x)^5$

$\sin x^5 = \sin(x^5)$