$$f_3(x,y) = e^{x^2+xy}$$

$$\frac{\partial f_3}{\partial x} = e^{x^2+xy}\cdot(2x+y)$$

$$\frac{\partial f_3}{\partial y} = e^{x^2+xy}\cdot(x)$$

$$g_1(x,y,z) = \sqrt{x^2+xyz} = (x^2+xyz)^{\frac{1}{2}}$$

$$\frac{\partial g_1}{\partial x} = \frac{1}{2}(x^2+xyz)^{-\frac{1}{2}}(2x+yz)$$

$$\frac{\partial g_1}{\partial y} = \frac{1}{2}(x^2+xyz)^{-\frac{1}{2}}\cdot \underline{xz}\ 1$$

$$\frac{\partial g_1}{\partial z} = \frac{1}{2}(x^2+xyz)^{-\frac{1}{2}}\cdot xy$$

$$h(x) = (x^2+1)^{\frac{1}{2}} = \sqrt{x^2+1}$$

$$h'(x) = \frac{1}{2}(x^2+1)^{-\frac{1}{2}}\cdot 2x$$

$$f_1(x) = 5 \qquad f_1'(x) = 0$$

$$f_2(x) = 5x^2 \qquad f_2'(x) = 5\cdot 2x$$

$$g_2(x,y,z) = \frac{x^2 \sin(x)\, y z}{z^2+4} = x^2 \sin(x)\, y \cdot \frac{z}{z^2+4}$$

konst.

$$\frac{\partial g_2}{\partial x} = \frac{yz}{z^2+4}\left(2x\sin x + x^2 \cos x\right)$$

konst.

$$\frac{\partial g_2}{\partial y} = \frac{x^2 \sin x \cdot z}{z^2+4} \cdot 1$$

konst.

$$\frac{\partial g_2}{\partial z} = x^2 \sin(x) \cdot y \, \frac{z^2+4 - z\cdot 2z}{(z^2+4)^2}$$

Spočítejte gradient v bodě A:

$h_1(x,y) = xy^2+5$, $A = [0,1]$

$\frac{\partial h_1}{\partial x} = y^2$ $\qquad$ $\frac{\partial h_1(A)}{\partial x} = 1^2 = 1$

$\frac{\partial h_1}{\partial y} = 2xy$ $\qquad$ $\frac{\partial h_1(A)}{\partial y} = 2 \cdot 0 \cdot 1 = 0$

$\operatorname{grad} h_1 = \left(\frac{\partial h_1}{\partial x}, \frac{\partial h_1}{\partial y}\right) = (y^2, 2xy)$

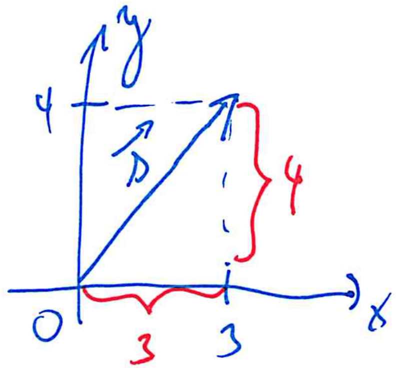
$\operatorname{grad} h_1(A) = (1, 0)$

$$h_3(x,y,z) = x^2 y + y^2 z + x y^2 \qquad A = [1,0,-1]$$

$$\left.\begin{array}{ll}
\dfrac{\partial h_3}{\partial x} = 2xy + y^2 & \dfrac{\partial h_3(A)}{\partial x} = 0 \\[2ex]
\dfrac{\partial h_3}{\partial y} = x^2 + 2yz + 2xy & \dfrac{\partial h_3(A)}{\partial y} = 1 \\[2ex]
\dfrac{\partial h_3}{\partial z} = y^2 & \dfrac{\partial h_3(A)}{\partial z} = 0
\end{array}\right\} \boxed{\operatorname{grad} h_3(A) = (0,1,0)}$$

Spočítejte směrovou derivaci v A ve směru $\vec{s}$

1) $f_1(x,y) = 2x^3y^2 + 5y \quad , \; A = [1,1], \; \vec{s} = (3,4)$



$$\frac{\partial f_1(A)}{\partial \vec{s}} = \operatorname{grad} f_1(A) \cdot \vec{s}_0 = (6,9) \cdot \left(\frac{3}{5}, \frac{4}{5}\right) = 6 \cdot \frac{3}{5} + 9 \cdot \frac{4}{5} = \frac{18+36}{5} = \underline{\underline{\frac{54}{5}}}$$

$$\left.\begin{aligned} \frac{\partial f_1}{\partial x} &= 6x^2y^2 & \frac{\partial f_1(A)}{\partial x} &= 6 \\ \frac{\partial f_1}{\partial y} &= 4x^3y + 5 & \frac{\partial f_1(A)}{\partial y} &= 9 \end{aligned}\right\} \operatorname{grad} f_1(A) = (6,9)$$

$$\|\vec{s}\| = \sqrt{3^2 + 4^2} = \sqrt{25} = \underline{5}$$

$$\vec{s}_0 = \frac{1}{\|\vec{s}\|}\vec{s} = \frac{1}{5}(3,4) = \underline{\underline{\left(\frac{3}{5}, \frac{4}{5}\right)}}$$

$$f_2(x,y) = y^2 \sin(xy) \quad , \quad A = [\pi, 2], \vec{s} = (1,1)$$

$$\frac{\partial f_2}{\partial x} = y^2 \cos(xy) \cdot y = y^3 \cos(xy) \qquad \frac{\partial f_2(A)}{\partial x} = 8$$

$$\frac{\partial f_2}{\partial y} = 2y \sin(xy) + y^2 \cos(xy) \cdot x \qquad \frac{\partial f_2(A)}{\partial y} = 4 \cdot 0 + 4 \cdot 1 \pi = 4\pi$$

$$\operatorname{grad} f_2(A) = (8, 4\pi)$$

$$\|\vec{s}\| = \sqrt{1^2 + 1^2} = \sqrt{2} \longrightarrow \vec{s}_0 = \frac{1}{\sqrt{2}}(1,1) = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$$

$$\frac{\partial f_2(A)}{\partial \vec{s}} = (8, 4\pi) \cdot \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right) = \frac{8}{\sqrt{2}} + \frac{4\pi}{\sqrt{2}}$$

$$f_3(x,y,z) = x^2 y + y^2 z + x z^2 \qquad A = [1,1,0], \quad \vec{s} = (-1,1,3)$$

$$\frac{\partial f_3}{\partial x} = 2xy + z^2 \qquad \frac{\partial f_3(A)}{\partial x} = 2$$

$$\frac{\partial f_3}{\partial y} = x^2 + 2yz \qquad \frac{\partial f_3(A)}{\partial y} = 1$$

$$\frac{\partial f_3}{\partial z} = y^2 + 2xz \qquad \frac{\partial f_3(A)}{\partial z} = 1$$

$$\operatorname{grad} f_3(A) = (2,1,1)$$

$$\|\vec{s}\| = \sqrt{1+1+9} = \sqrt{11}$$

$$\vec{s_0} = \frac{1}{\sqrt{11}}(-1,1,3)$$

$$\frac{\partial f(A)}{\partial \vec{s}} = (2,1,1) \cdot \frac{1}{\sqrt{11}}(-1,1,3) = \frac{1}{\sqrt{11}}(-2+1+3) = \underline{\frac{2}{\sqrt{11}}}$$

Pomocí diferenciálu odhadněte $0{,}997^2 \cdot 3{,}004^4$

$0{,}997^2 \cdot 3{,}004^4 \doteq 1^2 \cdot 3^4 = \underline{81}$ $\qquad A = [1,3]$

$f(x,y) = x^2 y^4$ $\qquad \underline{f(1,3) = 81}$ $\qquad 1 \to 0{,}997$

$h_x = -0{,}003$

dx

Δx

$f(0{,}997;\ 3{,}004) = ?$

dy

Δy

$3 \to 3{,}004$

$$df(A) = \frac{\partial f(A)}{\partial x} \cdot h_x + \frac{\partial f(A)}{\partial y} \cdot h_y = \boxed{162 h_x + 108 h_y}$$

$h_y = 0{,}004$

$$df(A) = 162 \cdot (-0{,}003) + 108 \cdot 0{,}004 = -0{,}486 + 0{,}432 = \underline{-0{,}054}$$

změna x-ové souřadnice

změna y-ové souřadnice

$$\frac{\partial f}{\partial x} = 2xy^4 \to \frac{\partial f(A)}{\partial x} = 162$$

$$\frac{\partial f}{\partial y} = 4x^2y^3 \to \frac{\partial f(A)}{\partial y} = 4 \cdot 27 = 108$$

$$f(0{,}997;\ 3{,}004) \doteq 81 - 0{,}054 = \underline{80{,}946}$$

$$\frac{2x^3+x^2+4x+1}{x^4-1} = \frac{A}{x+1} + \frac{B}{x-1} + \frac{Cx+D}{x^2+1} =$$

$$(x+1)(x-1)$$

$$= \frac{A(x-1)(x^2+1)+B(x+1)(x^2+1)+(Cx+D)(x^2-1)}{x^4-1} =$$

$$= \frac{A(x^3-x^2+x-1)+B(x^3+x^2+x+1)+Cx^3-Cx+Dx^2-D}{x^4-1}$$

$$u\,x^3:\ 2 = A+B+C$$

$$u\,x^2:\ 1 = -A+B+D$$

$$u\,x:\ 4 = A+B-C \quad (-1)$$

$$u\,x^0:\ 1 = -A+B-D \quad (-1)$$

$$-2 = 2C \rightarrow \boxed{C=-1}$$

$$0 = 2D \rightarrow \boxed{D=0}$$

$$3 = 2B-1 \rightarrow \boxed{B=2}$$

$$2 = A+2-1 \rightarrow \boxed{A=1}$$