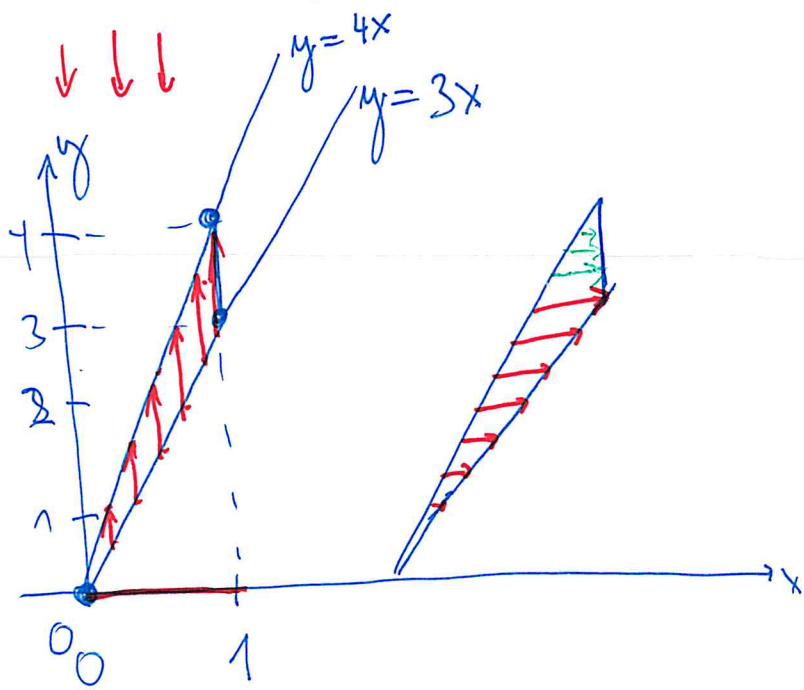


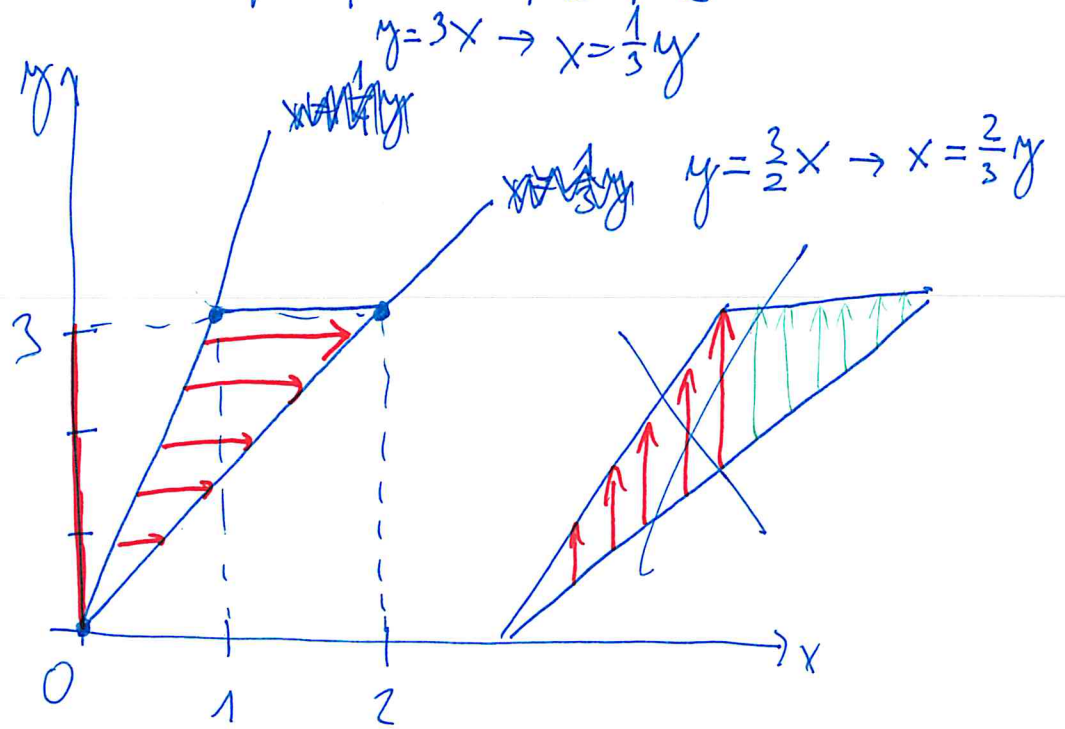
$$\triangle [0,0], [1,3], [1,4]$$



$$0 \leq x \leq 1$$

$$3x \leq y \leq 4x$$

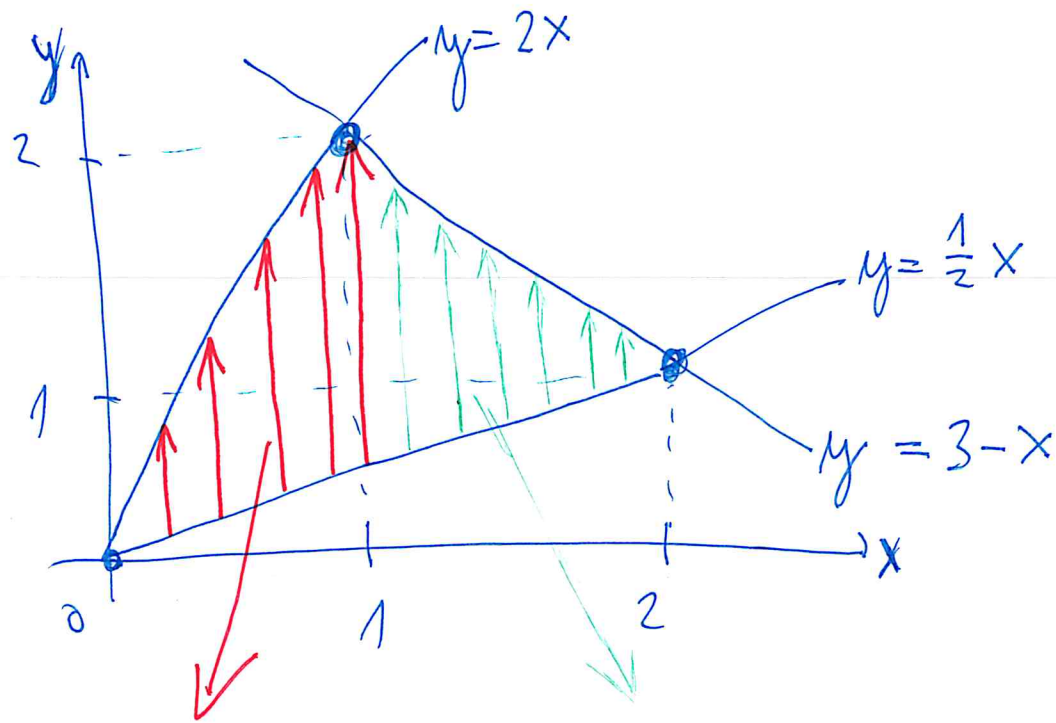
$$\triangle [0,0], [1,3], [2,3]$$



$$0 \leq y \leq 3$$

$$\frac{1}{3}y \leq x \leq \frac{2}{3}y$$

$$B: \Delta [0,0], [1,2], [2,1]$$



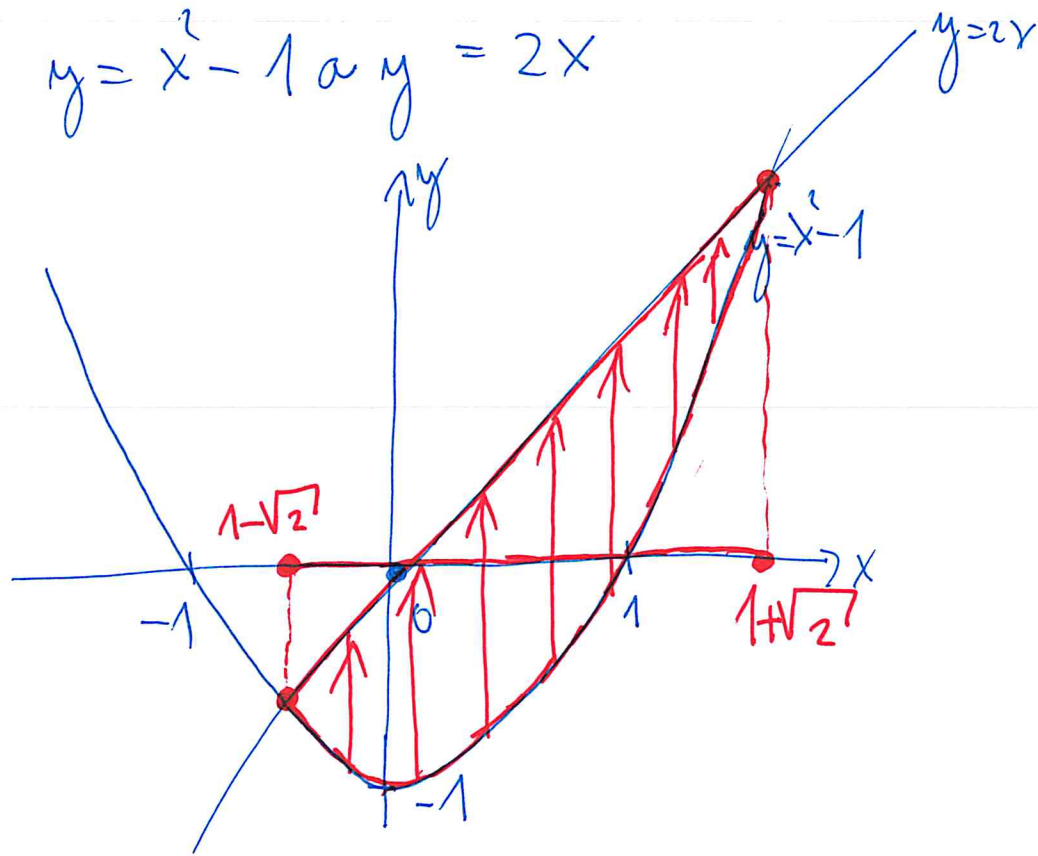
$$0 \leq x \leq 1$$

$$1 \leq x \leq 2$$

$$\frac{1}{2}x \leq y \leq 2x$$

$$\frac{1}{2}x \leq y \leq 3-x$$

$$3) \quad y = x^2 - 1 \quad \text{and} \quad y = 2x$$



$$1 - \sqrt{2} \leq x \leq 1 + \sqrt{2}$$

$$x^2 - 1 \leq y \leq 2x$$

$$x^2 - 1 = 2x$$

$$x^2 - 2x - 1 = 0$$

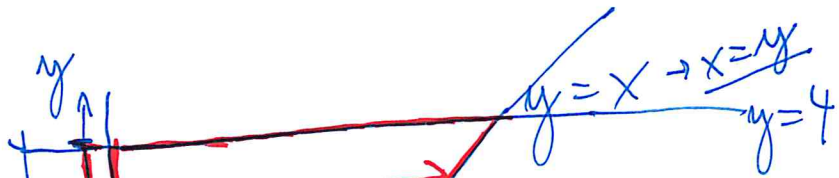
$$x_{1,2} = \frac{2 \pm \sqrt{8}}{2} = \frac{2 \pm \sqrt{2 \cdot 4}}{2} = \frac{2 \pm 2\sqrt{2}}{2}$$

$$x_{1,2} = \underline{\underline{1 \pm \sqrt{2}}} \quad \begin{array}{l} 1 + \sqrt{2} = 2.4 \\ 1 - \sqrt{2} = -0.4 \end{array}$$

$$4) \quad y = \frac{1}{x}, \quad y = x, \quad y = 4$$

$$\frac{1}{x} = x \quad | \cdot x$$

$$1 = x^2$$

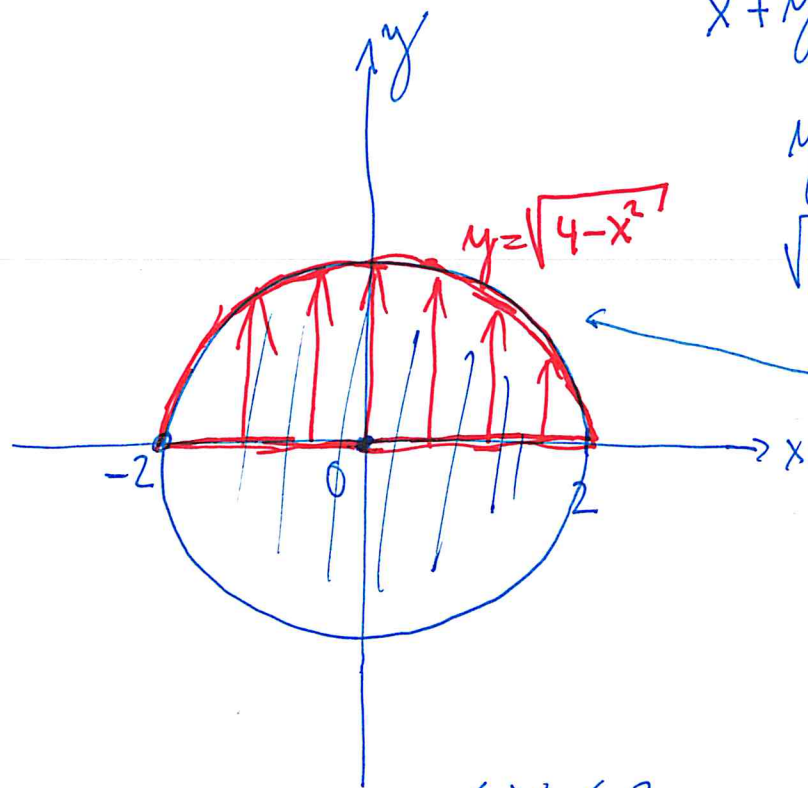


$$y = \frac{1}{x} \rightarrow x = \frac{1}{y}$$

$$1 \leq y \leq 4$$

$$\frac{1}{y} \leq x \leq y$$

$$5) \quad x^2 + y^2 \leq 4, \quad y \geq 0$$



$$-2 \leq x \leq 2$$

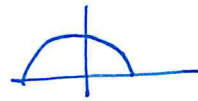
$$0 \leq y \leq \sqrt{4-x^2}$$

$$x^2 + y^2 = 4$$

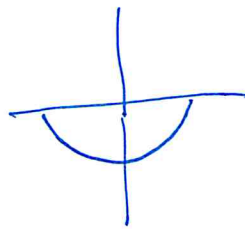
$$y^2 = 4 - x^2 \quad / \sqrt{\quad}$$

$$\sqrt{y^2} = \sqrt{4-x^2}$$

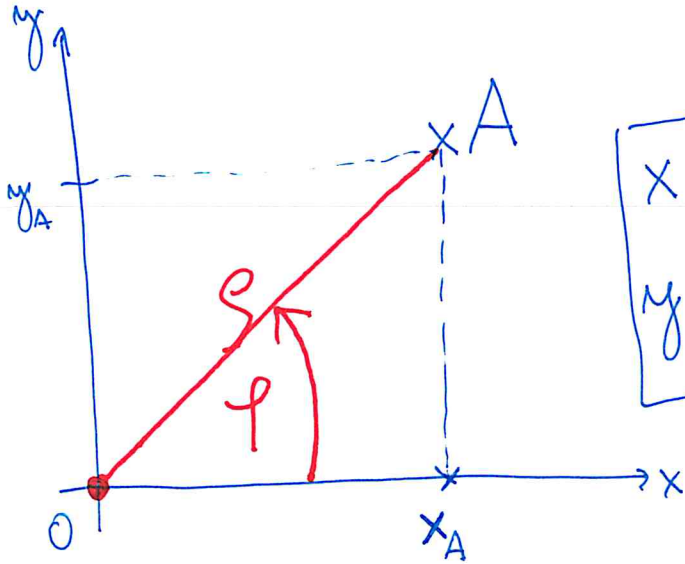
$$y = \sqrt{4-x^2}$$



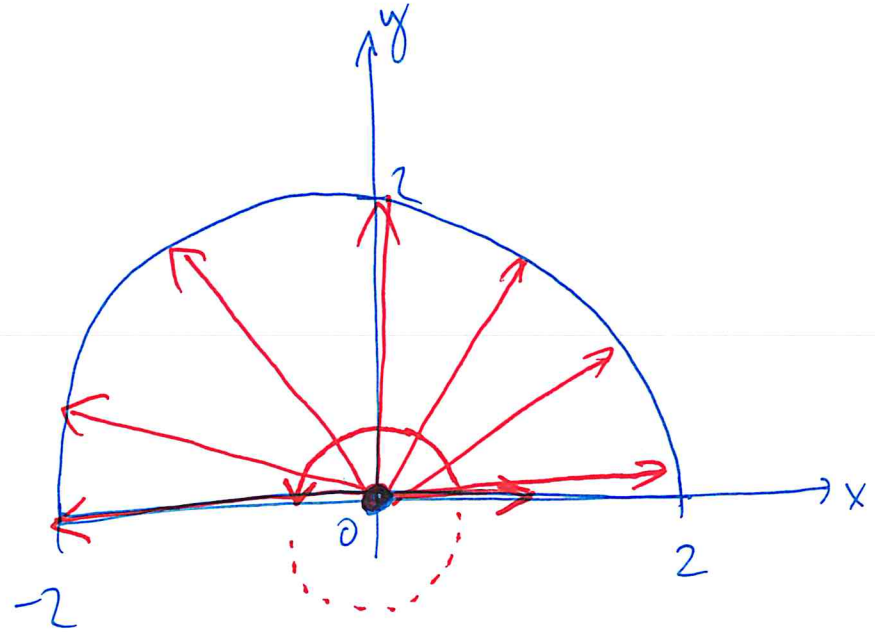
$$y = -\sqrt{4-x^2}$$



Polarne współrzędne



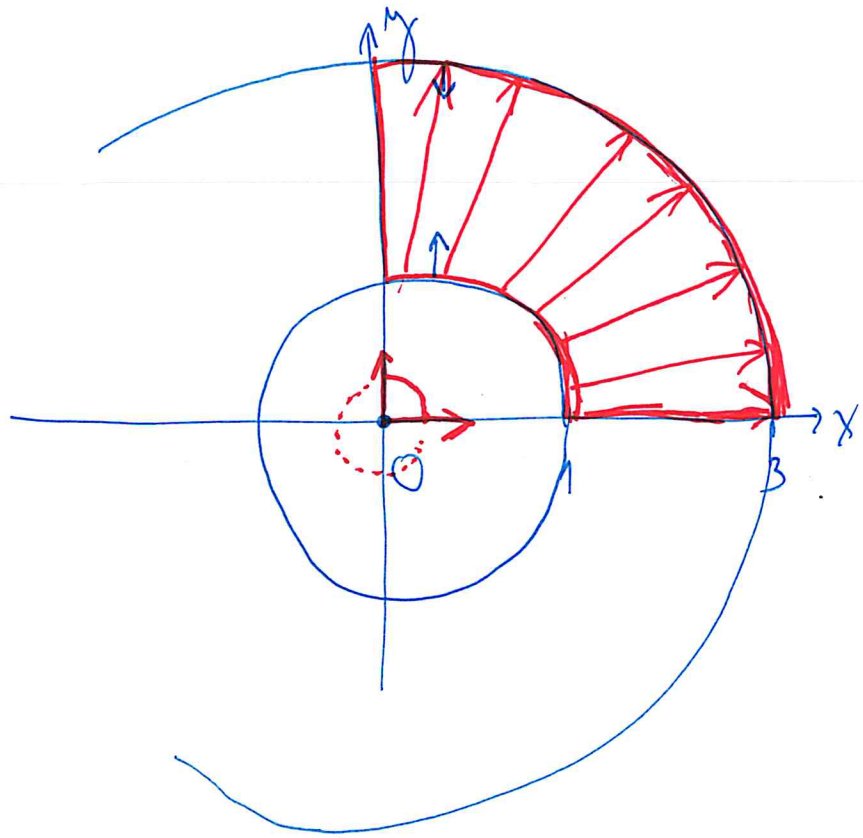
$$\begin{aligned}x &= \rho \cos \varphi \\y &= \rho \sin \varphi\end{aligned}$$



$$0 \leq \varphi \leq \pi$$

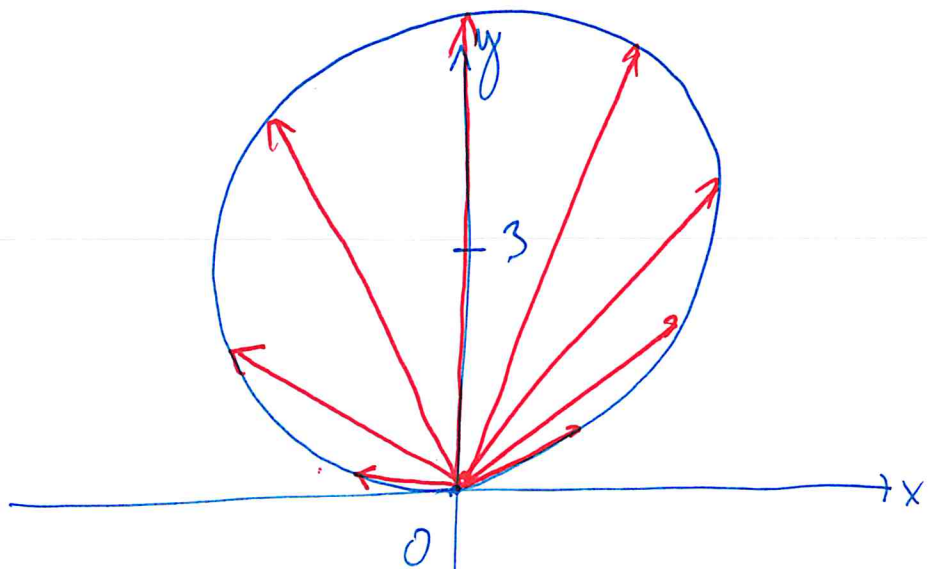
$$0 \leq \rho \leq 2$$

$$2) F : x^2 + y^2 \geq 1, x^2 + y^2 \leq 9, x \geq 0, y \geq 0$$



$$0 \leq \varphi \leq \frac{\pi}{2}$$

$$1 \leq \rho \leq 3$$



$$0 \leq \varphi \leq \pi$$

$$0 \leq \rho \leq \underline{\underline{6 \sin \varphi}}$$

$$\left. \begin{aligned} x &= \rho \cos \varphi \\ y &= \rho \sin \varphi \end{aligned} \right\}$$

$$x^2 + (y-3)^2 = 9$$

$$x^2 + y^2 - 6y + 9 = 9$$

$$x^2 + y^2 - 6y = 0$$

$$\rho^2 \cos^2 \varphi + \rho^2 \sin^2 \varphi - 6 \rho \sin \varphi = 0$$

$$\rho^2 (\underbrace{\cos^2 \varphi + \sin^2 \varphi}_1) - 6 \rho \sin \varphi = 0$$

$$\rho^2 - 6 \rho \sin \varphi = 0$$

$$\rho \cdot (\rho - 6 \sin \varphi) = 0$$

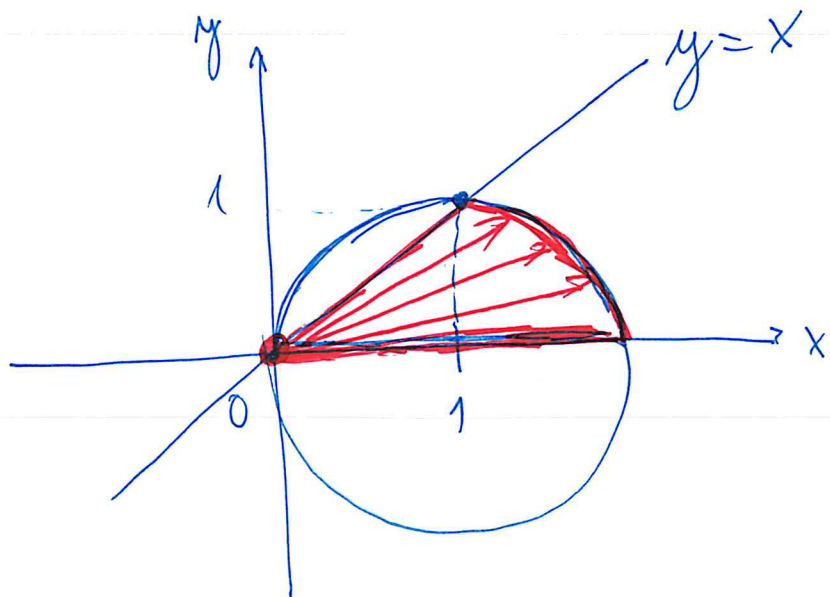
$$\rho = 0$$

$$\rho - 6 \sin \varphi = 0$$

$$\rho = 6 \sin \varphi$$

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4)



$$0 \leq \varphi \leq \frac{\pi}{4}$$

$$0 \leq \rho \leq 2\cos\varphi$$

$$\left. \begin{aligned} x &= \rho \cos\varphi \\ y &= \rho \sin\varphi \end{aligned} \right\}$$

$$(x-1)^2 + y^2 = 1$$

$$x^2 - 2x + 1 + y^2 = 1$$

$$x^2 + y^2 - 2x = 0$$

$$\rho^2 \cos^2\varphi + \rho^2 \sin^2\varphi - 2\rho \cos\varphi = 0$$

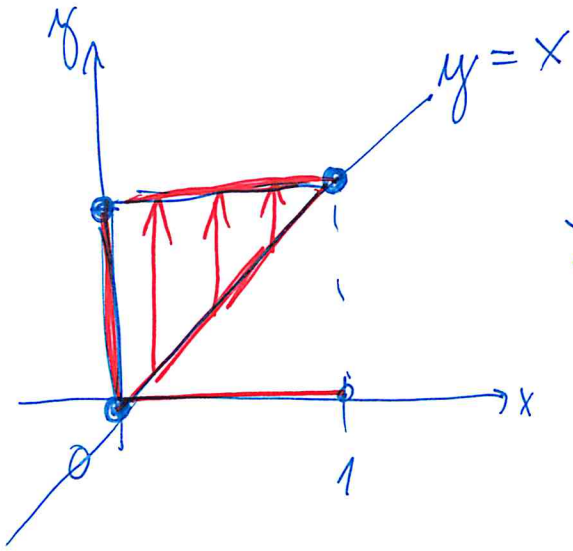
$$\rho^2 (\underbrace{\cos^2\varphi + \sin^2\varphi}_1) - 2\rho \cos\varphi = 0$$

$$\rho(\rho - 2\cos\varphi) = 0$$

$$\rho = 0 \vee \rho - 2\cos\varphi = 0 \rightarrow \rho = 2\cos\varphi$$

$$\iint_A 2xy \, dx \, dy = \int_0^1 \left[\int_x^1 2xy \, dy \right] dx = \int_0^1 (x - x^3) \, dx = \left[\frac{x^2}{2} - \frac{x^4}{4} \right]_0^1 = \frac{1}{2} - \frac{1}{4} = \underline{\underline{\frac{1}{4}}}$$

A je $\Delta [0,0], [0,1], [1,1]$

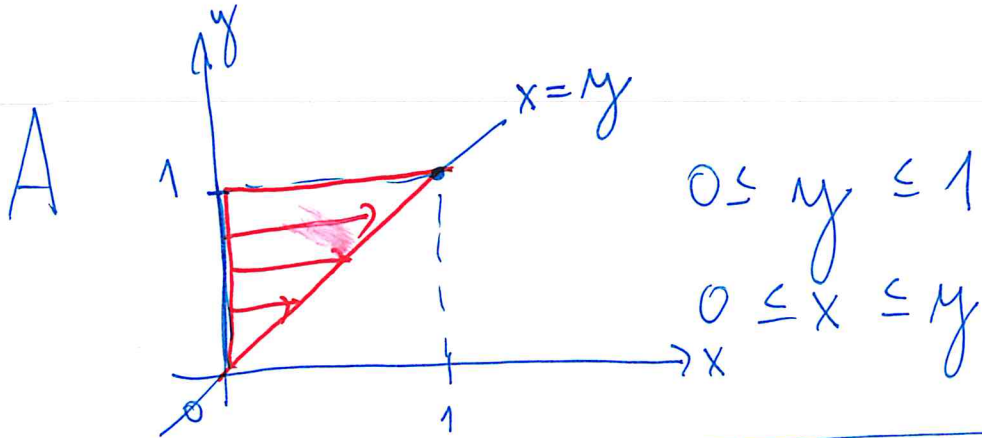


$$0 \leq x \leq 1$$

$$x \leq y \leq 1$$

$$*) \int_x^1 2xy \, dy = 2x \left[\frac{y^2}{2} \right]_x^1 = x(1 - x^2) = x - x^3$$

$$\iint_A 2xy \, dx \, dy = \int_0^1 \left[\int_0^y 2xy \, dx \right] dy = \int_0^1 y^3 \, dy = \left[\frac{y^4}{4} \right]_0^1 = \underline{\underline{\frac{1}{4}}}$$



$$*) \int_0^y 2xy \, dx = 2y \left[\frac{x^2}{2} \right]_0^y = y \cdot (y^2 - 0) = y^3$$

~~$$\iint_A 2xy \, dx \, dy = \int_0^y \left[\int_0^1 2xy \, dx \right] dy = \int_0^y x \, dx = \frac{1}{2} \left[x^2 \right]_0^y = \frac{1}{2} y^2 \quad ??$$~~

~~$$*) \int_0^1 2xy \, dy = 2x \left[\frac{y^2}{2} \right]_0^1 = x$$~~