Spočítejte objem $z = x^2+y^2$ a $z = 2x$

$z = \rho^2$

$$V = \iiint_A 1\,dx\,dy\,dz$$

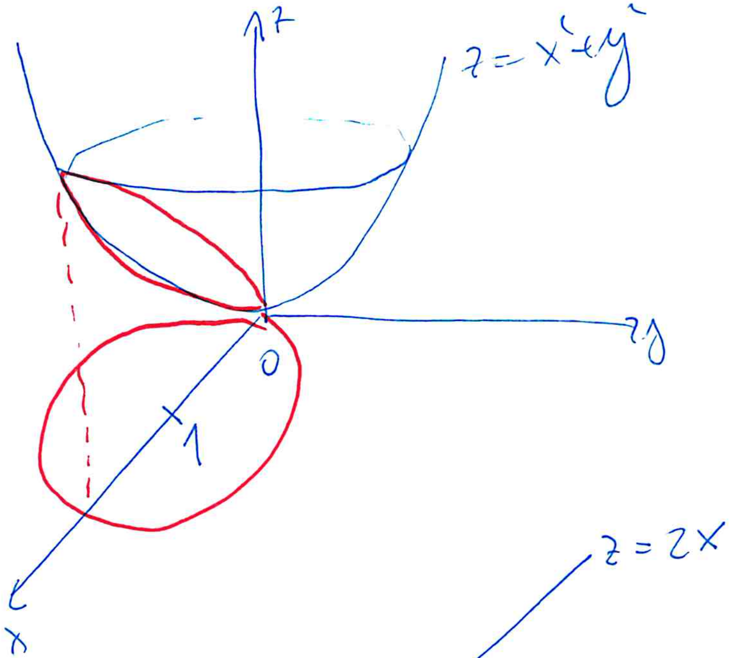
průsečík: $x^2+y^2 = 2x \longrightarrow \rho^2 = 2\rho\cos\varphi$

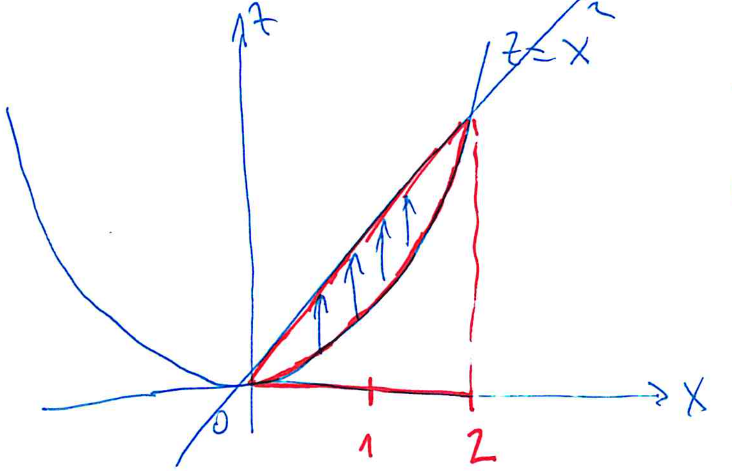
$$\boxed{\rho = 2\cos\varphi}$$

$x^2-2x+y^2 = 0$

$(x-1)^2+y^2 = 1$

$x^2-2x+1+y^2 = 1$

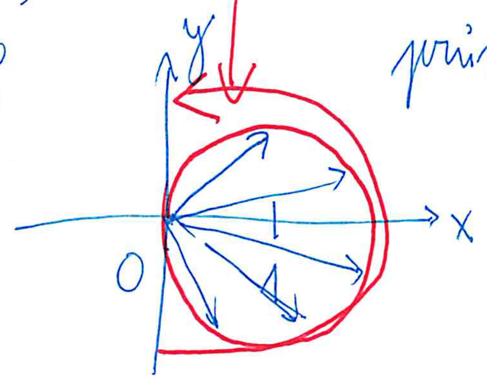




$x = \rho\cos\varphi$

$y = \rho\sin\varphi$

průmět



$$\boxed{\begin{aligned} -\tfrac{\pi}{2} &\le \varphi \le \tfrac{\pi}{2} \\ 0 &\le \rho \le 2\cos\varphi \\ \rho^2 &\le z \le 2\rho\cos\varphi \end{aligned}}$$

$$V = \iiint_A 1\,dx\,dy\,dz = \iiint_A \varrho\, d\varrho\, d\varphi\, dz = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left[\int_0^{2\cos\varphi} \left(\int_{\varrho^2}^{2\varrho\cos\varphi} \varrho\, dz \right) d\varrho \right] d\varphi =$$

$-\frac{\pi}{2} \le \varphi \le \frac{\pi}{2}$

$0 \le \varrho \le 2\cos\varphi$

$\varrho^2 \le z \le 2\varrho\cos\varphi$

$$= \frac{4}{3} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^4\varphi\, d\varphi = \frac{4}{\cancel{3}} \cdot \frac{\cancel{3}\pi}{8} = \boxed{\frac{\pi}{2} = V}$$

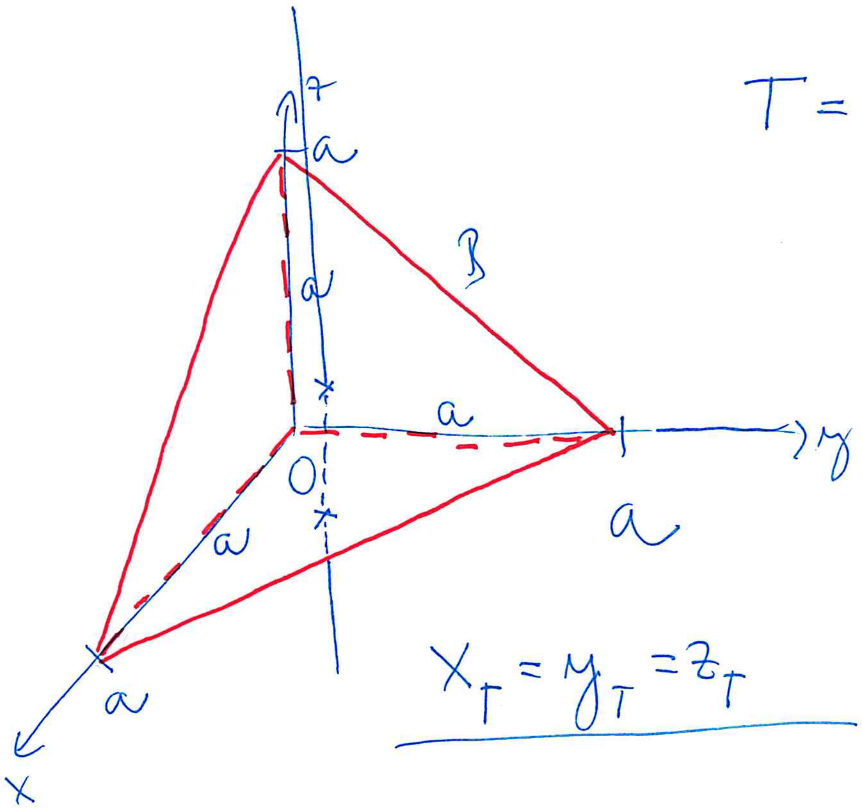
$$\int_{\varrho^2}^{2\varrho\cos\varphi} \varrho\, dz = \varrho\,[z]_{\varrho^2}^{2\varrho\cos\varphi} = \varrho\,(2\varrho\cos\varphi - \varrho^2) = 2\varrho^2\cos\varphi - \varrho^3$$

$$\boxed{\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^4\varphi\, d\varphi = \frac{3\pi}{8}}$$

$$\int_0^{2\cos\varphi} (2\varrho^2\cos\varphi - \varrho^3)\, d\varrho = \left[2\cos\varphi\,\frac{\varrho^3}{3} - \frac{\varrho^4}{4} \right]_0^{2\cos\varphi} = \frac{16}{3}\cos^4\varphi - 4\cos^4\varphi = \frac{4}{3}\cos^4\varphi$$

Těžiště tělesa $h(x,y,z)=1$ $\quad x+y+z=a,\ x=0,\ y=0,\ z=0$

$$z = a - x - y$$



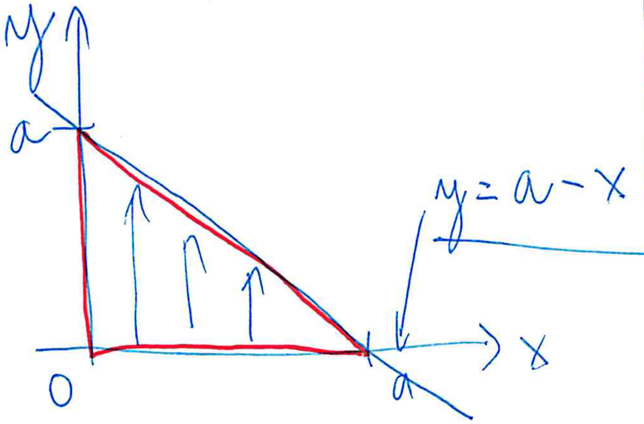
$$T = [x_T, y_T, z_T]$$

$$x_T = \frac{1}{m}\iiint_B x \, dx\,dy\,dz$$

$$z_T = \frac{1}{m}\iiint_B z \, dx\,dy\,dz$$

$$y_T = \frac{1}{m}\iiint_B y \, dx\,dy\,dz$$

$$x_T = y_T = z_T$$



$$\boxed{\begin{aligned} 0 &\le x \le a \\ 0 &\le y \le a - x \\ 0 &\le z \le a - x - y \end{aligned}}$$

$$m = h \cdot V = 1 \cdot \frac{1}{3} v \cdot S = \frac{1}{3} a \frac{1}{2} a^2 = \underline{\underline{\frac{1}{6} a^3}}$$

$$x_T = \frac{6}{a^3}\iiint_B x\,dx\,dy\,dz = \frac{6}{a^3}\int_0^a \left[\int_0^{a-x}\left(\int_0^{a-x-y} x\,dz\right)dy\right]dx \overset{3)}{=} \frac{6}{a^3}\int_0^a \left(\frac{1}{2}a^2x - ax^2 + \frac{1}{2}x^3\right)dx$$

$0 \le x \le a$

$0 \le y \le a - x$

$0 \le z \le a - x - y$

$$= \frac{6}{a^3}\left[\frac{1}{4}a^2x^2 - a\frac{x^3}{3} + \frac{x^4}{8}\right]_0^a = \frac{6}{a^3}\left(\frac{1}{4}a^4 - \frac{1}{3}a^4 + \frac{1}{8}a^4\right) =$$

$$= \frac{6}{\cancel{a^3}}\,\frac{\overbrace{6-8+3}^{1}}{24}\,a^{\cancel{4}} = \frac{1}{4}a$$

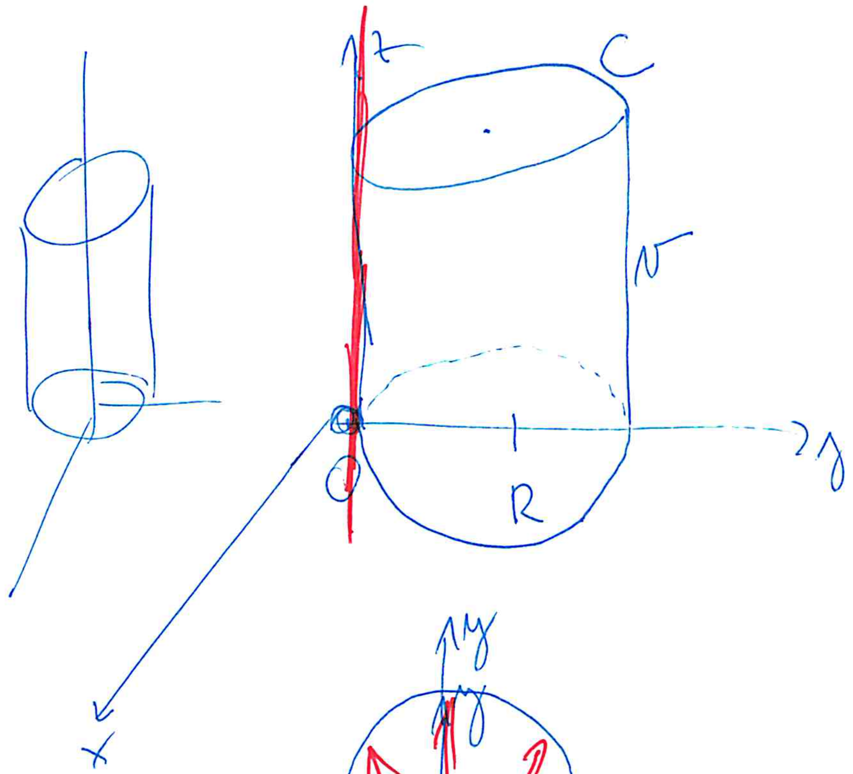
1) $$\int_0^{a-x-y} x\,dz = x\left[z\right]_0^{a-x-y} = x(a-x-y) = ax - x^2 - xy$$

$$\boxed{x_T = y_T = z_T = \frac{1}{4}a}$$

2) $$\int_0^{a-x} (ax - x^2 - xy)\,dy = \left[axy - x^2y - \frac{1}{2}xy^2\right]_0^{a-x} = ax(a-x) - x^2(a-x) - \frac{1}{2}x\cdot$$

$$\cdot(a^2 - 2ax + x^2) = a^2x - ax^2 - ax^2 + x^3 - \frac{1}{2}xa^2 + ax^2 - \frac{1}{2}x^3 = \frac{1}{2}a^2x - ax^2 + \frac{1}{2}x^3$$

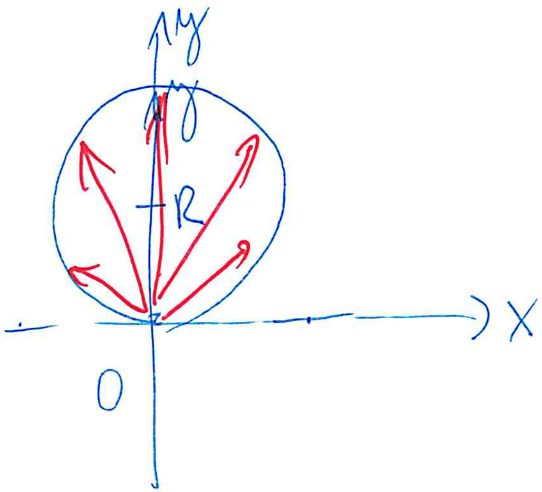
Moment setrvačnosti válce: R, v $\qquad h(x,y,z)=1$



$$I_z = \iiint_C (x^2+y^2)\, dx\,dy\,dz$$

$$\boxed{\begin{aligned} 0 &\le \varphi \le \pi \\ 0 &\le \rho \le 2R\sin\varphi \\ 0 &\le z \le v \end{aligned}}$$

$x=\rho\cos\varphi$
$y=\rho\sin\varphi$
$z=z$



$$x^2+(y-R)^2=R^2$$
$$x^2+y^2-2yR+\cancel{R^2}=\cancel{R^2}$$
$$x^2+y^2-2Ry=0$$
$$\rho^2-2R\rho\sin\varphi=0$$
$$\rho^2=2\rho\sin\varphi$$

$\rho=0$

$\boxed{\rho=2R\sin\varphi}$

$$I_z = \iiint_C (x^2+y^2)\,dx\,dy\,dz = \iiint_C \underbrace{\rho^2 \cdot \rho}_{\rho^3}\,d\rho\,d\varphi\,dz = \int_0^v \left[\int_0^\pi \left(\int_0^{2R\sin\varphi} \rho^3\,d\rho \right) d\varphi \right] dz \overset{3)}{=}$$

$$= \int_0^v \frac{3}{2}\pi R^4\,dz = \frac{3}{2}\pi R^4 \left[z\right]_0^v = \boxed{\frac{3}{2}\pi v R^4 = I_z}$$

$$0 \le \varphi \le \pi$$

$$0 \le \rho \le 2R\sin\varphi$$

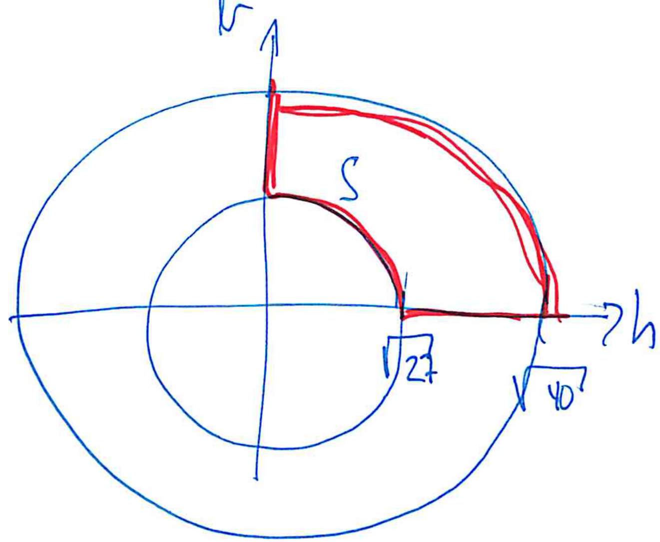
$$0 \le z \le v$$

1) $$\int_0^{2R\sin\varphi} \rho^3\,d\rho = \frac{1}{4}\left[\rho^4\right]_0^{2R\sin\varphi} = 4R^4\sin^4\varphi$$

2) $$\int_0^\pi 4R^4\sin^4\varphi\,d\varphi = 4R^4 \cdot \frac{3\pi}{8} = \frac{3}{2}\pi R^4$$

$$\int_0^\pi \sin^4\varphi\,d\varphi = \frac{3\pi}{8}$$

$$\iint_S \frac{9h^2+4b}{h^2+b^2}\,dh\,db = \iint_S \frac{9\rho^2\cos^2\varphi + 4\rho\sin\varphi}{\rho^2}\,\rho\,d\rho\,d\varphi =$$



$S:$

$$h^2+b^2 \geq 27$$
$$h^2+b^2 \leq 40$$
$$h \geq 0$$
$$b \geq 0$$

$$h = \rho\cos\varphi$$
$$b = \rho\sin\varphi$$

$$\sqrt{27} \leq \rho \leq \sqrt{40}$$
$$0 \leq \varphi \leq \frac{\pi}{2}$$

$$= \iint_S \left(9\rho\cos^2\varphi + 4\sin\varphi\right) d\rho\,d\varphi = \int_0^{\frac{\pi}{2}} \left(\int_{\sqrt{27}}^{\sqrt{40}} \left(9\rho\cos^2\varphi + 4\sin\varphi\right) d\rho \right) d\varphi$$