$$y'' - 3y' - 10y = 5e^{2x}\cos 0x$$

2) $\alpha = 2$, $\beta = 0$ $\Big\}$ $\alpha + i\beta = 2 + i0 = 2$

$k = 0$

$0\times$

1) $y'' - 3y' - 10y = 0$

$$\lambda^2 - 3\lambda - 10 = 0$$

$$(\lambda - 5)(\lambda + 2) = 0$$

$$\lambda_1 = 5$$

$$\lambda_2 = -2$$

$$y_H = c_1 e^{-2x} + c_2 e^{5x}$$

$c_1, c_2 \in \mathbb{R}$

$$y_p = \underbrace{e^{2x}}_{} \cdot \underbrace{x^0}_{=1} \Big(A \cdot \underbrace{\cos 0x}_{=1} + B \underbrace{\sin 0x}_{=0}\Big) = A e^{2x}$$

$$y_p' = A\,2e^{2x} \qquad y_p'' = 4Ae^{2x}$$

dosazení: $4A\cancel{e^{2x}} - 6A\cancel{e^{2x}} - 10A\cancel{e^{2x}} = 5\cancel{e^{2x}}$

$$-12A = 5 \rightarrow A = -\frac{5}{12}$$

$$y_p = -\frac{5}{12}e^{2x}$$

obecné řešení: $\boxed{y = y_H + y_p = c_1 e^{-2x} + c_2 e^{5x} - \frac{5}{12}e^{2x}}$

$$y'' + 9y = 1 \cdot \sin 3x \, e^{0x}, \quad y(0)=0, \quad y'(0)=0$$

1) $y'' + 9y = 0$

$$\lambda^2 + 9 = 0$$
$$\lambda^2 = -9$$
$$\lambda_{1,2} = 0 \pm 3i$$

$$y_1 = e^{0x} \cos 3x = \cos 3x$$
$$y_2 = e^{0x} \sin 3x = \sin 3x$$

$$y_H = c_1 \cos 3x + c_2 \sin 3x$$
$$c_1, c_2 \in \mathbb{R}$$

2) $\left.\begin{aligned} \alpha &= 0 \\ \beta &= 3 \end{aligned}\right\} \alpha + i\beta = 0 + 3i = 3i$

$k = 1$

$1x$

$$y_p = e^{0x} \cdot x^1 (A \cos 3x + B \sin 3x)$$
$$y_p = x (A \cos 3x + B \sin 3x) = -\frac{1}{6} x \cos 3x$$

$$y_p' = (A\cos 3x + B \sin 3x) + x(-3A \sin 3x + 3B \cos 3x)$$
$$y_p'' = -3A \sin 3x + 3B\cos 3x + (-3A \sin 3x + 3B \cos 3x) +$$
$$+ x(-9A \cos 3x - 9B \sin 3x)$$

dosazení: $-6A \sin 3x + 6B \cos 3x + \cancel{x(-9A\cos 3x - 9B \sin 3x)} +$
$$+ \cancel{9x(A\cos 3x + B \sin 3x)} = \sin 3x$$

$\sin x: -6A = 1 \rightarrow A = -\frac{1}{6} \quad \cos x: 6B = 0 \rightarrow B = 0$

obecné řešení: $y = y_H + y_P = C_1 \cos 3x + C_2 \sin 3x - \frac{1}{6} x \cos 3x$

$y' = -3C_1 \sin 3x + 3C_2 \cos 3x - \frac{1}{6}\cos 3x + \frac{1}{6} x 3 \sin 3x$

3) $y(0)=0$, $y'(0)=0$: $0 = C_1 + \cancel{C_2 \cdot 0} - \cancel{\frac{1}{6}\cdot 0} \rightarrow \underline{C_1 = 0}$

$0 = 3C_2 - \frac{1}{6} \rightarrow 3C_2 = \frac{1}{6} \rightarrow C_2 = \underline{\underline{\frac{1}{18}}}$

$$\boxed{y = \frac{1}{18}\sin 3x - \frac{1}{6} x \cos 3x}$$

$$y'''' - 4y'' = 80x^3$$

1) $$y'''' - 4y'' = 0$$

$$\lambda^4 - 4\lambda^2 = 0$$

$$\lambda^2(\lambda^2 - 4) = 0$$

$$(\lambda - 0)^2(\lambda - 2)(\lambda + 2) = 0$$

$$\lambda_{1,2} = 0 \quad \lambda_3 = 2 \quad \lambda_4 = -2$$

$$y_H = C_1 + C_2 x + C_3 e^{2x} + C_4 e^{-2x}$$

2) $$\left.\begin{array}{l}\alpha = 0\\ \beta = 0\end{array}\right\} \alpha + i\beta = 0 + 0i = 0$$

$$K = 2$$

$$y_p = \underbrace{e^{0x}}_{=1} \cdot x^2 \left[(Ax^3 + Bx^2 + Cx + D) \cdot \underbrace{\cos 0x}_{=1} + (Ex^3 + Fx^2 + Gx + H) \underbrace{\sin 0x}_{=0} \right]$$

$$y_p = Ax^5 + Bx^4 + Cx^3 + Dx^2$$

$$y_p' = 5Ax^4 + 4Bx^3 + 3Cx^2 + 2Dx$$

$$y_p'' = 20Ax^3 + 12Bx^2 + 6Cx + 2D$$

$$y_p''' = 60Ax^2 + 24Bx + 6C$$

$$y_p'''' = 120Ax + 24B$$

dosazení: $120Ax + 24B - \cancel{4}\,80Ax^3 - 48Bx^2 - 24Cx - 8D = 80x^3$

u x^3: $-80A = 80 \rightarrow A = -1$

u x^2: $-48B = 0 \rightarrow B = 0$

u x: $120A - 24C = 0 \rightarrow 24C = 120A = -120 \rightarrow C = -\frac{120}{24} = -5$

u x^0: $24B - 8D = 0 \rightarrow D = 0$

$y_p = -x^5 - 5x^3$

obecné řešení: $y = y_H + y_p = \underline{c_1 + c_2 x + c_3 e^{2x} + c_4 e^{-2x} - x^5 - 5x^3}$

$\underline{c_1, c_2, c_3, c_4 \in \mathbb{R}}$

$$y'' - 2y' + 2y = e^x \sin x \quad , y(0) = \frac{1}{2} , y'(0) = 0$$

1) $y'' - 2y' + 2y = 0$

$$\lambda^2 - 2\lambda + 2 = 0$$

$$\lambda_{1,2} = \frac{2 \pm \sqrt{-4}}{2} = \frac{2 \pm 2i}{2} = 1 \pm i$$

$$y = c_1 e^x \cos x + c_2 e^x \sin x$$

2) $\alpha = 1$ $\quad \alpha + i\beta = 1 + 1i$

$\beta = 1$

$k = 1$

$$y_p = (e^x \cdot x)(A\cos x + B\sin x)$$

$$y_p' = (e^x \cdot x + e^x)(A\cos x + B\sin x) + (e^x \cdot x)(-A\sin x + B\cos x)$$

$$y_p'' = (e^x \cdot x + e^x + e^x)(A\cos x + B\sin x) + (e^x \cdot x + e^x)(-A\sin x + B\cos x) +$$

$$+ (e^x \cdot x + e^x)(-A\sin x + B\cos x) + (e^x \cdot x)(-A\cos x - B\sin x)$$

$$\cancel{e^x\cdot x A\cos x} + \cancel{e^x\cdot x\cdot B\sin x} + \cancel{2e^x A\cos x} + \cancel{2e^x B\sin x} - \cancel{A\sin x\cdot e^x\cdot x} + \cancel{B\cos x\, e^x\cdot x}$$

$$- e^x A\sin x + e^x\cdot B\cos x - \cancel{A\sin x\, e^x\cdot x} + \cancel{B\cos x\, e^x\cdot x} - A\sin x\, e^x + B\cos x\, e^x$$

$$-\cancel{A\cos x\, e^x\cdot x} - \cancel{B\sin x\, e^x\cdot x} - 2\cancel{A\cos x\, e^x\cdot x} - \cancel{2B\sin x\, e^x\cdot x} - \cancel{2Ae^x\cos x} - \cancel{2Be^x\sin x}$$

$$+\cancel{2A\sin x\, e^x\cdot x} - \cancel{2B\cos x\, e^x\cdot x} + \cancel{2A\cos x\cdot x e^x} + \cancel{2B\sin x\, e^x\cdot x} = e^x\sin x$$

$\cancel{-2A\sin x\, e^x = e^x\sin x}$ $\to$: u $\quad \sin x\cdot e^x:\ -2A = 1 \to A = -\frac{1}{2}$

$\cos x\cdot e^x:\ 2B = 0 \to B = 0$

obecné řešení

$$\boxed{y = y_H + y_p = c_1 e^x\cos x + c_2 e^x\sin x - \frac{1}{2}(x e^x)\cos x} \qquad y_p = -\frac{1}{2}x\cdot e^x\cos x$$

$$y' = c_1 e^x\cos x - c_1 e^x\sin x + c_2 e^x\sin x + c_2 e^x\cos x - \frac{1}{2}(e^x + x e^x)\cos x + \frac{1}{2}x e^x\sin x$$

$$\frac{1}{2} = c_1$$

$$0 = c_1 + c_2 - \frac{1}{2} \to c_2 = 0$$

$$\boxed{y = \frac{1}{2}e^x\cos x - \frac{1}{2}x e^x\cos x}$$