1) Nalezněte odchylku vektorů z $\mathbb{R}^4$: $\vec{a} = (1,0,0,1)$

$\vec{b} = \left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right)$

$$\boxed{\cos\varphi = \frac{\vec{a}\cdot\vec{b}}{\|\vec{a}\|\cdot\|\vec{b}\|}} = \frac{1}{\sqrt{2}\cdot 1} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} \to \boxed{\varphi = \frac{\pi}{4}}$$

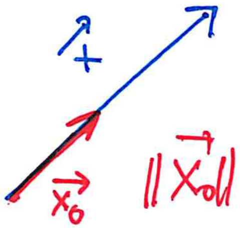
$$\vec{a}\cdot\vec{b} = (1,0,0,1)\cdot\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right) = 1\cdot\frac{1}{2} + 0\cdot\frac{1}{2} + 0\cdot\frac{1}{2} + 1\cdot\frac{1}{2} = 1$$

$$\boxed{\|\vec{a}\| = \sqrt{\vec{a}\cdot\vec{a}}} = \sqrt{1^2+0^2+0^2+1^2} = \sqrt{2}$$

$$\|\vec{b}\| = \sqrt{\vec{b}\cdot\vec{b}} = \sqrt{\left(\frac{1}{2}\right)^2+\left(\frac{1}{2}\right)^2+\left(\frac{1}{2}\right)^2+\left(\frac{1}{2}\right)^2} = \sqrt{1} = 1$$ (je jednotkový vektor)

2) V $\mathbb{R}^3$ nalezněte jednotkové vektory stejného směru a orientace jako $\vec{x} = (0, -3, 4)$ resp. $\vec{z} = (2, 2, 1)$.

$$\|\vec{x}\| = \sqrt{\vec{x}\cdot\vec{x}} = \sqrt{0^2 + (-3)^2 + 4^2} = \sqrt{25} = 5$$



$\|\vec{x}_0\| = 1$

$$\vec{x}_0 = \alpha\cdot\vec{x} = \boxed{\frac{1}{\|\vec{x}\|}\cdot\vec{x}} = \frac{1}{5}\vec{x} = \frac{1}{5}(0, -3, 4) = \left(0, -\frac{3}{5}, \frac{4}{5}\right)$$

$$\|\vec{x}_0\| = \sqrt{\vec{x}_0\cdot\vec{x}_0} = \sqrt{0^2 + \left(-\frac{3}{5}\right)^2 + \left(\frac{4}{5}\right)^2} = \sqrt{\frac{9}{25} + \frac{16}{25}} = \sqrt{\frac{25}{25}} = \underline{\underline{1}}$$

$$\boxed{\vec{z}_0 = \frac{1}{\|\vec{z}\|}\vec{z}} = \frac{1}{3}\vec{z} = \frac{1}{3}(2, 2, 1) = \left(\frac{2}{3}, \frac{2}{3}, \frac{1}{3}\right)$$

$$\|\vec{z}\| = \sqrt{\vec{z}\cdot\vec{z}} = \sqrt{2^2 + 2^2 + 1^2} = \sqrt{9} = 3$$

3) V $\mathbb{R}^5$ nalezněte vektory kolmé současně $\vec{u}=(0,1,3,2,1)$

$$\vec{v}=(1,1,0,0,1)$$

~~odchylka $\vec{u}$ a $\vec{v}$:~~ $\cancel{\cos\varphi = \frac{\vec{u}\cdot\vec{v}}{\quad}}$

odchylka $\vec{u}$ a $\vec{x}$:

$$\cos\varphi = \frac{\overbrace{\vec{u}\cdot\vec{x}}^{=0}}{\|\vec{u}\|\cdot\|\vec{x}\|} = \cos\frac{\pi}{2} = 0$$

$$\boxed{\begin{aligned}\vec{u}\cdot\vec{x} &= 0\\ \vec{v}\cdot\vec{x} &= 0\end{aligned}}\quad\begin{aligned}&(\vec{x} \text{ je kolmé na } \vec{u})\\ &(\vec{x} \text{ je kolmé na } \vec{v})\end{aligned}$$

$$\vec{x}=(a,b,c,d,e)$$

$$\vec{u}\cdot\vec{x}=0$$

$$\vec{v}\cdot\vec{x}=0$$

$$\vec{u}=(0,1,3,2,1)$$

$$\vec{v}=(1,1,0,0,1)$$

$$\vec{x}=(a,b,c,d,e)$$

$$(0,1,3,2,1)\cdot(a,b,c,d,e)=0 \to \boxed{\begin{aligned} b+3c+2d+e&=0 \\ a+b+e&=0 \end{aligned}} \to$$

$$(1,1,0,0,1)\cdot(a,b,c,d,e)=0 \to$$

$$e=t\ (t\in\mathbb{R})$$

$$d=s\ (s\in\mathbb{R})$$

$$c=r\ (r\in\mathbb{R})$$

$$b=-3r-2s-t$$

$$a=3r+2s+\cancel{t}-\cancel{t}$$

$$\vec{x}=\begin{bmatrix} a \\ b \\ c \\ d \\ e \end{bmatrix}=\begin{bmatrix} 3r+2s \\ -3r-2s-t \\ r \\ s \\ t \end{bmatrix}=\begin{bmatrix} 3r \\ -3r \\ r \\ 0 \\ 0 \end{bmatrix}+\begin{bmatrix} 2s \\ -2s \\ 0 \\ s \\ 0 \end{bmatrix}+\begin{bmatrix} 0 \\ -t \\ 0 \\ 0 \\ t \end{bmatrix}=$$

$$=r\begin{bmatrix} 3 \\ -3 \\ 1 \\ 0 \\ 0 \end{bmatrix}+s\begin{bmatrix} 2 \\ -2 \\ 0 \\ 1 \\ 0 \end{bmatrix}+t\begin{bmatrix} 0 \\ -1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

řešení má dimensi 3

4)
$$\begin{bmatrix} 1 & 2 & 1 & 3 \\ 1 & 0 & 2 & 5 \end{bmatrix} \begin{matrix} (-1) \\ \leftarrow \end{matrix} \sim \begin{bmatrix} 1 & 2 & 1 & 3 \\ 0 & -2 & 1 & 2 \end{bmatrix}$$

$$\begin{aligned} a+2b+c+3d&=0 \\ -2b+c+2d&=0 \end{aligned}$$

$d = t \ (t \in \mathbb{R})$

$b = s \ (s \in \mathbb{R})$

$c = 2s - 2t$

$a = -2s - 2s + 2t - 3t$

$a = -t - 4s$

$$\vec{x} = \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} -t-4s \\ s \\ 2s-2t \\ t \end{bmatrix} = \begin{bmatrix} -t \\ 0 \\ -2t \\ t \end{bmatrix} + \begin{bmatrix} -4s \\ s \\ 2s \\ 0 \end{bmatrix} = t\begin{bmatrix} -1 \\ 0 \\ -2 \\ 1 \end{bmatrix} + s\begin{bmatrix} -4 \\ 1 \\ 2 \\ 0 \end{bmatrix}$$

Řešení má dimenzi 2.

$$\boxed{A \cdot \vec{x} = \vec{o}}$$

$$\begin{bmatrix} 1 & 1 & 2 & 1 & -3 \\ 1 & 1 & 2 & 1 & 0 \\ 2 & 2 & 4 & 2 & -3 \\ 4 & 4 & 8 & 4 & -6 \end{bmatrix} \begin{matrix} (-1)\,(-2)\,(-4) \end{matrix} \sim \begin{bmatrix} 1 & 1 & 2 & 1 & -3 \\ 0 & 0 & 0 & 0 & 3 \\ \cancel{0} & \cancel{0} & \cancel{0} & \cancel{0} & \cancel{3} \\ \cancel{0} & \cancel{0} & \cancel{0} & \cancel{0} & \cancel{6} \end{bmatrix}$$

$a+b+2c+d-3e=0$

$3e=0$

$e=0$

$d=t\ (t\in\mathbb{R})$

$c=r\ (r\in\mathbb{R})$

$b=s\ (s\in\mathbb{R})$

$a=-s-2r-t$

$$\vec{x}=\begin{bmatrix} a \\ b \\ c \\ d \\ e \end{bmatrix}=\begin{bmatrix} -s-2r-t \\ s \\ r \\ t \\ 0 \end{bmatrix}=\begin{bmatrix} -s \\ s \\ 0 \\ 0 \\ 0 \end{bmatrix}+\begin{bmatrix} -2r \\ 0 \\ r \\ 0 \\ 0 \end{bmatrix}+\begin{bmatrix} -t \\ 0 \\ 0 \\ t \\ 0 \end{bmatrix}=$$

$$= s\begin{bmatrix} -1 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}+r\begin{bmatrix} -2 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}+t\begin{bmatrix} -1 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

řešení má dimenzi 3

5)
$$\left[\begin{array}{cccc|c} 1 & 2 & 1 & 3 & 7 \\ 1 & 0 & 2 & 5 & 11 \end{array}\right] \overset{(-1)}{} \sim \left[\begin{array}{cccc|c} 1 & 2 & 1 & 3 & 7 \\ 0 & -2 & 1 & 2 & 4 \end{array}\right]$$

$$a + 2b + c + 3d = 7$$
$$-2b + c + 2d = 4$$

$$d = t \ (t \in \mathbb{R})$$
$$b = s \ (s \in \mathbb{R})$$
$$c = 4 + 2s - 2t$$
$$a = 7 - 2s - 4 - 2s + 2t - 3t$$
$$a = 3 - 4s - t$$

$$\vec{x} = \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 3 - 4s - t \\ s \\ 4 + 2s - 2t \\ t \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \\ 4 \\ 0 \end{bmatrix} + \begin{bmatrix} -4s \\ s \\ 2s \\ 0 \end{bmatrix} + \begin{bmatrix} -t \\ 0 \\ 2t \\ t \end{bmatrix} =$$

$$= \underbrace{\begin{bmatrix} 3 \\ 0 \\ 4 \\ 0 \end{bmatrix}}_{\text{Partikulární řešení}} + \underbrace{t \begin{bmatrix} -1 \\ 0 \\ 2 \\ 1 \end{bmatrix} + s \begin{bmatrix} -4 \\ 1 \\ 2 \\ 0 \end{bmatrix}}_{\text{řešení přidružené homogenní soustavy}}$$