$$\int_0^1 \frac{1}{(5x+1)^3}\,dx = \left[-\frac{1}{10}\frac{1}{(5x+1)^2}\right]_0^1 = -\frac{1}{10}\frac{1}{36} - \left(-\frac{1}{10}\right)$$

$$\int \frac{1}{(5x+1)^3}\,dx = \frac{1}{5}\int \frac{1}{t^3}\,dt = \frac{1}{5}\int t^{-3}\,dt = \frac{1}{5}\,\frac{t^{-2}}{-2} + C =$$

$t = 5x+1$

$dt = 5\,dx \quad /:5$

$\frac{1}{5}\,dt = dx$

$$= -\frac{1}{10}\frac{1}{t^2} + C = -\frac{1}{10}\frac{1}{(5x+1)^2} + C$$

$$\frac{(5x+1)^{-2}}{-2}\cdot\frac{1}{5}$$

$$\int_0^1 \frac{1}{(5x+1)^3}\,dx = \frac{1}{5}\int_1^6 \frac{1}{t^3}\,dt = \frac{1}{5}\left[\frac{t^{-2}}{-2}\right]_1^6 = -\frac{1}{10}\left(\frac{1}{36}-1\right)$$

$$t = 5x+1$$

$$dt = 5dx$$

$$\frac{1}{5}dt = dx$$

$$\int \frac{x^4-5x^3+8x^2-6x+3}{x^2-5x+6}\,dx = \int\left(\underset{\checkmark}{x^2}+\underset{\checkmark}{2}+\underset{?}{\frac{4x-9}{x^2-5x+6}}\right)dx =$$

$$(x^4-5x^3+8x^2-6x+3):(x^2-5x+6) = x^2+2$$

$$\begin{array}{l}
-(x^4-5x^3+6x^2) \\
\hline
\qquad 2x^2-6x+3 \\
\quad -(2x^2-10x+12) \\
\hline
\qquad\qquad 4x-9
\end{array}$$

$$= \frac{x^3}{3}+2x+\ln|x-2|+3\ln|x-3|+c$$

$$\int \frac{4x-9}{x^2-5x+6}\,dx = \int \frac{4x-9}{(x-2)(x-3)}\,dx = \int \left(\frac{1}{x-2} + \frac{3}{x-3}\right) dx =$$

$$= \ln|x-2| + 3\ln|x-3| + C$$

$$x \neq \{2,3\}$$

$$x^2-5x+6=0$$

$$(x-2)(x-3)=0$$

$$\frac{4x\boxed{-9}}{(x-2)(x-3)} = \frac{\overset{1}{A}}{x-2} + \frac{\overset{3}{B}}{x-3} = \frac{A(x-3)+B(x-2)}{(x-2)(x-3)} = \frac{Ax-3A+Bx-2B}{(x-2)(x-3)} =$$

$$= \frac{(A+B)\cdot x\boxed{-3A-2B}}{(x-2)(x-3)}$$

$$x: \quad A+B=4 \quad /\cdot 2$$

$$x^0: \quad -3A-2B=-9$$

$$-A=-1 \rightarrow \boxed{\begin{aligned} A&=1 \\ B&=3 \end{aligned}}$$

$$\int \frac{4x^2+17x+13}{(x-3)(x+2)^2}\,dx = \int\left(\frac{4}{x-3}+\frac{1}{(x+2)^2}\right)dx = 4\ln|x-3| + \frac{(x+2)^{-1}}{-1} + C$$

$$x \neq \{3,-2\}$$

$$\frac{4x^2+17x+13}{(x-3)(x+2)^2} = \frac{\overset{4}{A}}{x-3} + \frac{\overset{1}{B}}{(x+2)^2} + \frac{\overset{0}{C}}{x+2} = \frac{A(x+2)^2+B(x-3)+C(x-3)(x+2)}{(x-3)(x+2)^2} =$$

$$= \frac{A(x^2+4x+4)+B(x-3)+C(x^2-x-6)}{(x-3)(x+2)^2}$$

$$u\ x^2:\ 4 = A + C \longrightarrow 4 = A + C \ /\cdot 9$$

$$\boxed{C=0}$$

$$u\ x:\ 17 = 4A + B - C \ /\cdot 3$$

$$u\ x^0:\ 13 = 4A - 3B - 6C$$

$$64 = 16A - 9C$$

$$100 = 25A \rightarrow \boxed{A=4}$$

$$\boxed{B=1}$$

$$a^2-b^2=(a-b)(a+b)$$

$$\int \frac{2x^3+x^2+4x+1}{x^4-1}\,dx = \int\left(\frac{1}{x+1}+\frac{2}{x-1}-\frac{x}{x^2+1}\right)dx = \ln|x+1|+2\ln|x-1|-\frac{1}{2}\ln(x^2+1)+c$$

$1 \quad 2 \quad -1 \quad 0$

$$\frac{2x^3+x^2+4x+1}{x^4-1}=\frac{2x^3+x^2+4x+1}{(x^2-1)(x^2+1)}=\frac{2x^3+x^2+4x+1}{(x+1)(x-1)(x^2+1)}=\frac{A}{x+1}+\frac{B}{x-1}+\frac{Cx+D}{x^2+1}=$$

$$=\frac{A(x-1)(x^2+1)+B(x+1)(x^2+1)+(Cx+D)(x+1)(x-1)}{(x+1)(x-1)(x^2+1)}$$

$x \neq -1, 1$

$$\int \frac{x}{x^2+1}\,dx = \frac{1}{2}\int \frac{1}{t}\,dt = \frac{1}{2}\ln t + c = \frac{1}{2}\ln(x^2+1)+c$$

$t = x^2+1$

$dt = 2x\,dx \quad /:2$

$\frac{1}{2}dt = x\,dx$

$$\int \frac{3x^2+7x}{(x-1)(x^2+4x+5)}\,dx = \int\left(\frac{1}{x-1} + \frac{2x+5}{x^2+4x+5}\right)dx = \ln|x-1| + \dots$$

$$\frac{3x^2+\boxed{7}x}{(x-1)(x^2+4x+5)} = \frac{\overset{1}{A}}{x-1} + \frac{\overset{2}{B}x+\overset{5}{C}}{x^2+4x+5} = \frac{A(x^2+4x+5)+(Bx+C)(x-1)}{(x-1)(x^2+4x+5)} =$$

$$= \frac{Ax^2+\boxed{4A}x+5A+Bx^2\boxed{-B}x+\boxed{C}x-C}{(x-1)(x^2+4x+5)}$$

$$\left.\begin{aligned} u\,x^2 &: 3 = A+B \\ u\,x &: 7 = 4A-B+C \\ u\,x^0 &: 0 = 5A-C \end{aligned}\right\} \quad \begin{aligned} 3 &= A+B \\ 7 &= 9A-B \\ \hline 10 &= 10A \rightarrow \boxed{A=1} \quad \boxed{B=2} \quad \boxed{C=5} \end{aligned}$$

$$\int \frac{2x+5}{x^2+4x+5}\,dx = \int \left(\frac{2x+4}{x^2+4x+5} + \frac{1}{x^2+4x+5} \right) dx$$

$$t = x^2+4x+5$$

$$dt = (2x+4)\,dx$$

$$\int \frac{\overbrace{(2x+4)\,dx}^{dt}}{\underbrace{x^2+4x+5}_{t}} = \int \frac{1}{t}\,dt = \ln t + c = \ln(x^2+4x+5) + c$$

$$= \int \frac{dt}{t}$$

$$\int \frac{1}{x^2+4x+5}\,dx = \int \frac{1}{(x+2)^2+1}\,dx = \int \frac{1}{r^2+1}\,dr = \operatorname{arctg}(x+2) + c$$

$$r = x+2$$

$$dr = dx$$

$$= \operatorname{arctg} r + c$$