$$\int_0^{\frac{\pi}{2}} (1-\sin 2x)\,dx = \left[x + \cos 2x \cdot \frac{1}{2}\right]_0^{\frac{\pi}{2}} = \frac{\pi}{2} + (-1)\cdot\frac{1}{2} - \left(\frac{1}{2}\right) = \underline{\frac{\pi}{2} - 1}$$

$A = 2x$

$\sin 2x = 2\sin x\cos x$

$$*) = \frac{\pi^2}{2} - 2\left[\sin x\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} = \frac{\pi^2}{2} - 2\left(1-(-1)\right) = \underline{\underline{\frac{\pi^2}{2} - 4}}$$

2

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} x^2\cdot\cos x\,dx = \left[x^2\sin x\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} - \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 2x\sin x\,dx = \left(\frac{\pi^2}{4}\cdot 1 - \left(-\frac{\pi}{2}\right)^2(-1)\right) - \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 2x\sin x\,dx =$$

$u = x^2 \qquad u' = 2x$

$v' = \cos x \qquad v = \sin x$

$$= \frac{\pi^2}{2} - \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 2x\sin x\,dx = \frac{\pi^2}{2} - \left(\left[-2x\cos x\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} + \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 2\cos x\,dx\right) =$$

0

$u = 2x \qquad u' = 2$

$v' = \sin x \qquad v = -\cos x$

$= *)$

$$\int_0^1 \frac{1}{x^2-4}\,dx = \int_0^1 \frac{1}{(x-2)(x+2)} = \int_0^1 \left(\frac{-\frac{1}{4}}{x+2} + \frac{\frac{1}{4}}{x-2} \right) dx = \left[-\frac{1}{4}\ln(x+2) \right]_0^1 +$$

$$+ \left[\frac{1}{4}\ln|x-2| \right]_0^1 = -\frac{1}{4}\left(\ln 3 - \ln 2\right) + \frac{1}{4}\left(\underbrace{\ln 1}_{=0} - \ln 2\right) =$$

$$\frac{1+0x}{(x+2)(x-2)} = \frac{A}{x+2} + \frac{B}{x-2} = \frac{A(x-2)+B(x+2)}{(x-2)(x+2)} = \frac{(A+B)x - 2A + 2B}{(x-2)(x+2)} =$$

$$= \frac{-\frac{1}{4}}{x+2} + \frac{\frac{1}{4}}{x-2}$$

$$u\,x: 0 = A + B \quad /\cdot 2$$

$$u\,x^0: 1 = -2A + 2B$$

$$1 = 4B \rightarrow B = \frac{1}{4} \quad A = -\frac{1}{4}$$

$$\rightarrow \underline{-\frac{1}{4}\ln 3} + \cancel{\frac{1}{4}\ln 2} - \cancel{\frac{1}{4}\ln 2}$$

$$\int_0^3 \frac{4x}{x^2+1}\,dx = 2\int_0^3 \frac{2x}{x^2+1}\,dx = 2\int_1^{10} \frac{dt}{t} = 2\left[\ln t\right]_1^{10} =$$

$$t = x^2+1$$

$$dt = 2x\,dx$$

$$= 2\ln 10 = \ln 10^2 = \ln 100$$

$$\int \frac{3x^2+4x+3}{(x^2+1)(x+1)^2}\,dx = \int \frac{2}{x^2+1} + \frac{1}{(x+1)^2}\,dx = 2\,\mathrm{arctg}\,x + \frac{(x+1)^{-1}}{-1} + C$$

$$\frac{3x^2+4x+3}{(x^2+1)(x+1)^2} = \frac{Ax+B}{x^2+1} + \frac{C}{(x+1)^2} + \frac{D}{x+1} = \frac{(Ax+B)(x^2+2x+1) + C(x^2+1) + D(x^2+1)(x+1)}{(x^2+1)(x+1)^2} =$$

$$= \frac{Ax^3+2Ax^2+Ax+Bx^2+2Bx+B+Cx^2+C+Dx^3+Dx^2+Dx+D}{(x^2+1)(x+1)^2}$$

$x^3: 0 = A+D$

$x^2: 3 = 2A+B+C+D$

$x: 4 = A+2B+D \rightarrow 4 = 2B \rightarrow \boxed{B=2}$

$x^0: 3 = B+C+D \;/\,(-1)$

$\left.\begin{matrix} 3 = 2A+B+C+D \\ 3 = B+C+D \end{matrix}\right\} 0 = 2A \rightarrow \boxed{\begin{matrix} A=0 \\ D=0 \end{matrix}}$

$3 = 2+C$

$\boxed{C=1}$

$$\int_0^{\frac{\pi}{2}} \cos^3 x\, dx = \int_0^{\frac{\pi}{2}} \cos^2 x \cdot \cos x\, dx = \int_0^{\frac{\pi}{2}} (1-\sin^2 x)\cos x\, dx = \int_0^{1} (1-t^2)\, dt =$$

$$\boxed{\sin^2 x + \cos^2 x = 1}$$

$$t = \sin x$$

$$dt = \cos x\, dx$$

$$x = 0 \to t = \sin 0 = 0$$

$$x = \frac{\pi}{2} \to t = \sin\frac{\pi}{2} = 1$$

$$= \left[t - \frac{t^3}{3} \right]_0^1 = 1 - \frac{1}{3} = \frac{2}{3}$$