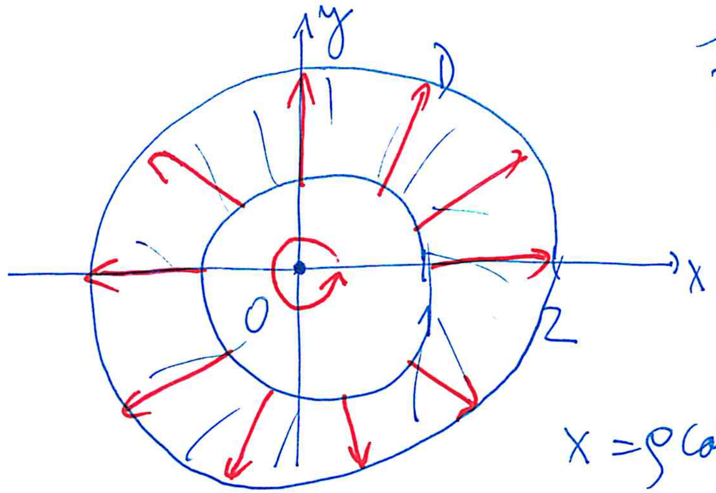
Moment setrvačnosti

$$I = \iint_D (x^2+y^2)\cdot h(x,y)\,dx\,dy = h\iint_D (x^2+y^2)\,dx\,dy = \boxed{h\cdot\frac{15}{2}\pi = I}$$

$x^2+y^2=1$

$x^2+y^2=4$

$h(x,y)=h$



$x=\rho\cos\varphi$

$y=\rho\sin\varphi$

$0\le\varphi\le 2\pi$

$1\le\rho\le 2$

$$\iint_D (x^2+y^2)\,dx\,dy = \iint_D (\rho^2\cos^2\varphi+\rho^2\sin^2\varphi)\rho\,d\rho\,d\varphi = \iint_D \rho^3\,d\rho\,d\varphi = \int_0^{2\pi}\left(\int_1^2 \rho^3\,d\rho\right)d\varphi =$$

$$= \int_0^{2\pi}\frac{1}{4}\left[\rho^4\right]_1^2 d\varphi = \frac{1}{4}\int_0^{2\pi}15\,d\varphi = \frac{15}{4}\left[\varphi\right]_0^{2\pi} = \frac{15}{2}\pi$$

Těžiště : $x^2+y^2=2x$ — a — $x^2+y^2=4x$

$h(x,y)=1$

$x^2-2x+y^2=0$

$(x-1)^2+y^2=1$

$x^2-4x+y^2=0$

$(x-2)^2+y^2=4$

$T=[x_T, y_T]$

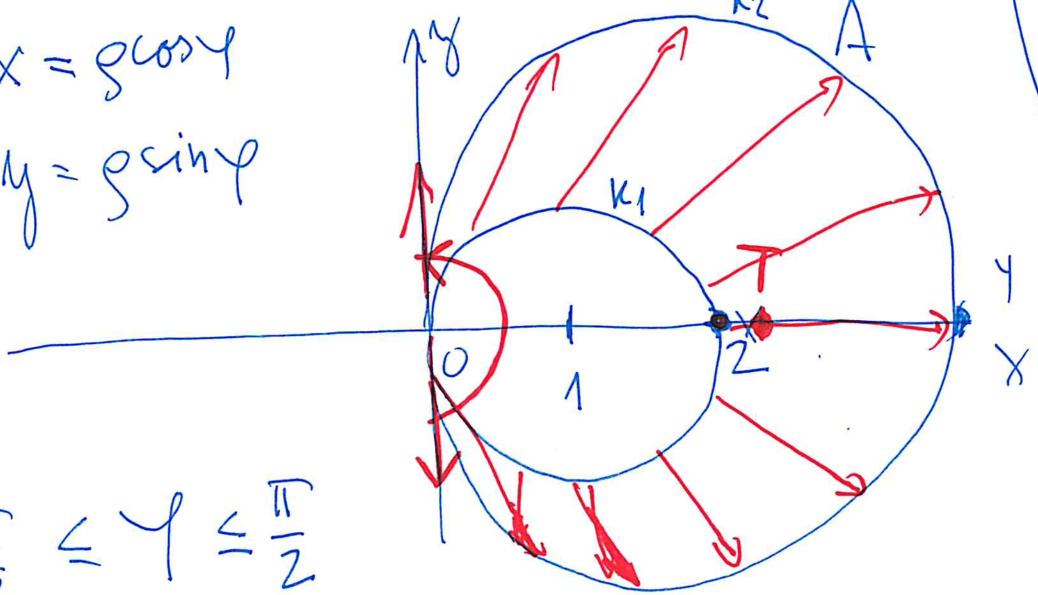
$m = h \cdot S = \pi \cdot 4 - \pi$

$\underline{m = 3\pi}$

$\boxed{y_T = 0}$

$x=\varrho\cos\varphi$

$y=\varrho\sin\varphi$



$$x_T = \frac{1}{m}\iint_A x \, dxdy = \frac{1}{3\pi}\iint_A x \, dxdy$$

$\varrho^2 = 2\varrho\cos\varphi$ $\qquad$ $\varrho^2 = 4\varrho\cos\varphi$

$k_1 : \boxed{\varrho = 2\cos\varphi}$ $\qquad$ $k_2 : \boxed{\varrho = 4\cos\varphi}$

$-\frac{\pi}{2} \le \varphi \le \frac{\pi}{2}$

$2\cos\varphi \le \varrho \le 4\cos\varphi$

$$x_T = \frac{1}{3\pi}\iint_A x\,dx\,dy = \frac{1}{3\pi}\iint_A \rho^2\cos\varphi\,d\rho\,d\varphi = \frac{1}{3\pi}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\left[\underline{\int_{2\cos\varphi}^{4\cos\varphi}\rho^2\cos\varphi\,d\rho}\right]d\varphi =$$

$$-\frac{\pi}{2} \le \varphi \le \frac{\pi}{2}$$

$$2\cos\varphi \le \rho \le 4\cos\varphi$$

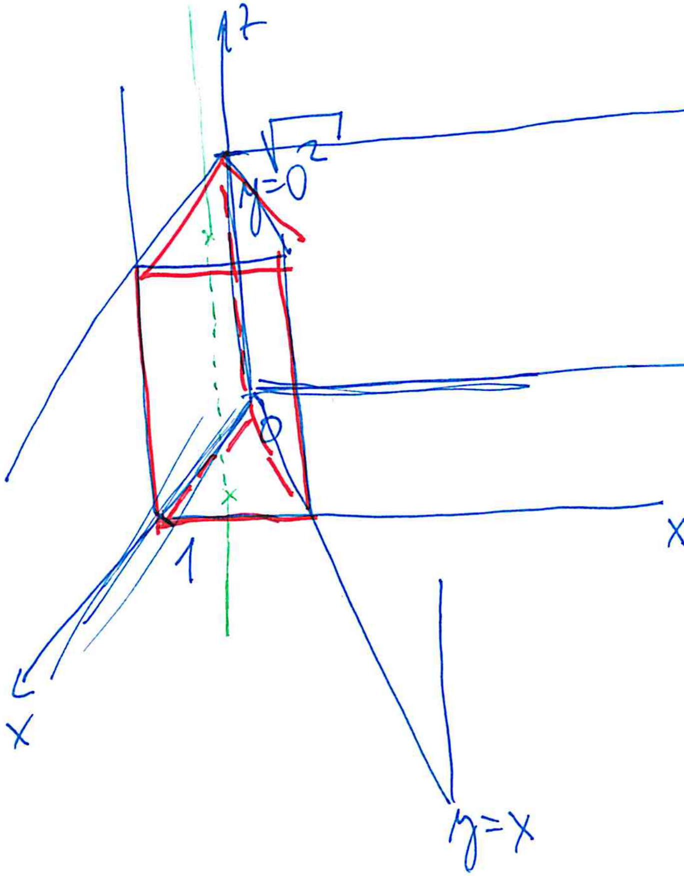
$$= \frac{1}{3\pi}\cdot\frac{56}{3}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\cos^4\varphi\,d\varphi = \frac{1}{3\cancel{\pi}}\cdot\frac{56}{\cancel{3}}\,\frac{\cancel{3}}{8}\cancel{\pi} = \boxed{\frac{7}{3} = x_T}$$

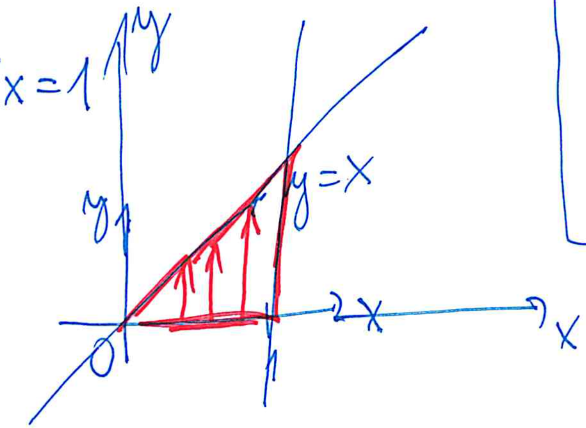
$$\int_{2\cos\varphi}^{4\cos\varphi}\rho^2\cos\varphi\,d\rho = \frac{1}{3}\cos\varphi\left[\rho^3\right]_{2\cos\varphi}^{4\cos\varphi} = \frac{1}{3}\cos\varphi\left(64\cos^3\varphi - 8\cos^3\varphi\right) = \underline{\frac{56}{3}\cos^4\varphi}$$

$$\iiint_A (x+y+z)\,dx\,dy\,dz = \int_0^1 \left[\int_0^x \left(\int_0^{\sqrt{2}} (x+y+z)\,dz \right) dy \right] dx =$$

$$= \frac{\sqrt{2}}{2} + \frac{1}{2}$$

$A: \; x=1, \; y=0, \; y=x, \; z=0, \; z=\sqrt{2}$





$$\boxed{\begin{aligned} 0 &\le x \le 1 \\ 0 &\le y \le x \\ 0 &\le z \le \sqrt{2} \end{aligned}}$$

1) $$\int_0^{\sqrt{2}} (x+y+z)\,dz = \left[xz + yz + \frac{1}{2}z^2\right]_0^{\sqrt{2}} = \sqrt{2}x + \sqrt{2}y + 1$$

2) $$\int_0^{x} (\sqrt{2}x + \sqrt{2}y + 1)\,dy = \left[\sqrt{2}xy + \sqrt{2}\frac{y^2}{2} + y\right]_0^{x} = \underline{\sqrt{2}x^2 + \frac{\sqrt{2}}{2}x^2 + x}$$

3) $$\int_0^{1} \left(\frac{3\sqrt{2}}{2}x^2 + x\right)dx = \left[\frac{\sqrt{2}}{2}x^3 + \frac{x^2}{2}\right]_0^{1} = \underline{\underline{\frac{\sqrt{2}}{2} + \frac{1}{2}}}$$

$$\iiint_B y^2\,dx\,dy\,dz = \int_0^8 \left[\int_0^{4-\frac{1}{2}x} \left(\int_0^{\frac{8-x-2y}{4}} y^2\,dz \right) dy \right] dx$$

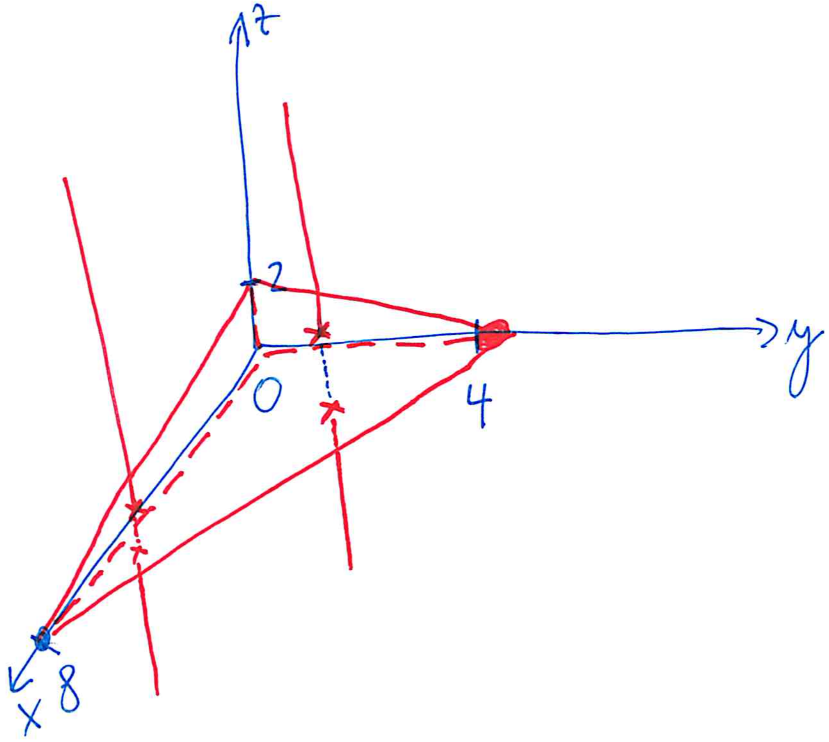
$B:\ x=0,\ y=0,\ z=0,\ x+2y+4z=8 \rightarrow 4z = 8-x-2y \ /:4$

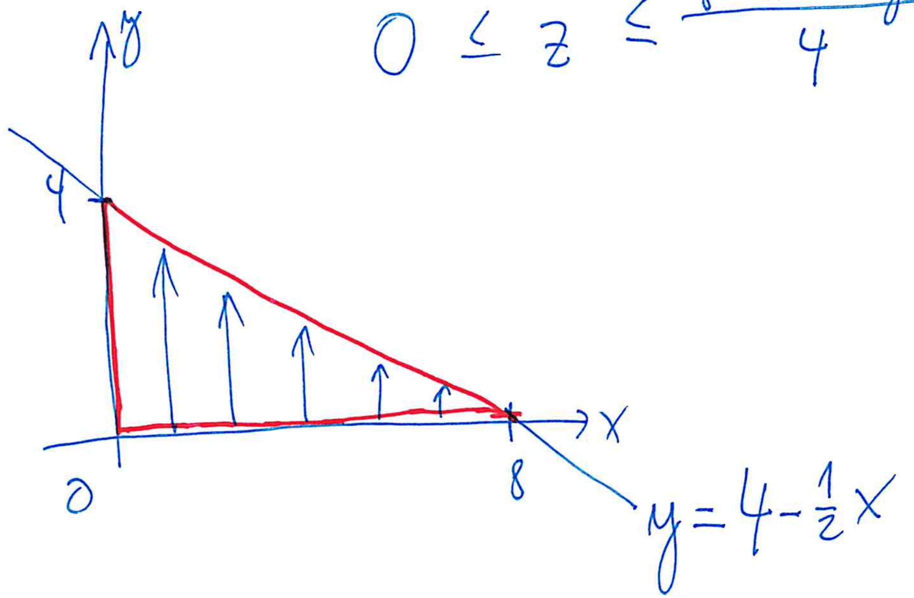
$$z = \frac{8-x-2y}{4}$$

$$0 \le x \le 8$$

$$0 \le y \le 4-\frac{1}{2}x$$

$$0 \le z \le \frac{8-x-2y}{4}$$





$$\int_0^{\frac{8-x-2y}{4}} y^2\,dz = y^2\,[z]_0^{\frac{8-x-2y}{4}} = y^2 \cdot \frac{8-x-2y}{4} = \frac14\left(8y^2 - xy^2 - 2y^3\right)$$

$$\frac14\int_0^{4-\frac12 x}\left(8y^2 - xy^2 - 2y^3\right)dy = \frac14\left(\left[\frac83 y^3 - \frac{x}{3}y^3 - \frac12 y^4\right]_0^{4-\frac12 x} = \frac14\left(-\frac13 x^3 + 8x^2 - 64x + \frac{512}{3} - (*)\right.\right.$$

$$\left(4-\frac12 x\right)^4 = \frac1{16}x^4 - 2x^3 + 24x^2 - 128x + 256$$

$$\left(4-\frac12 x\right)^3 = -\frac18 x^3 + 3x^2 - 24x + 64$$

$$(*)\ -\left(-\frac1{24}x^4 + x^3 - 8x^2 + \frac{64}{3}x\right) - \left(\frac1{32}x^4 - x^3 + 12x^2 - 64x + 128\right)\Bigg) = \frac1{384}x^4 - \frac1{12}x^3 + x^2 - \frac{16}{3}x + \frac{32}{3}$$

$$\int_0^8\left(\frac1{384}x^4 - \frac1{12}x^3 + x^2 - \frac{16}{3}x + \frac{32}{3}\right)dx = \left[\frac1{384}\frac{x^5}{5} - \frac1{64}x^4 + \frac{x^3}{3} - \frac{16}{6}x^2 + \frac{32}{3}x\right]_0^8 =$$

$$= \frac{256}{15}\ (?)$$

$$g(b,h,c) = \frac{3b^5 \cdot h^4}{5h^3 + 8c^2} = 3b^5h^4 \cdot (5h^3+8c^2)^{-1}$$

$$B = [5, 8, 9]$$

$$\frac{\partial g}{\partial b} = \frac{3h^4}{5h^3+8c^2} \, 5b^4$$

$$\frac{\partial g}{\partial h} =$$

$$\frac{\partial g}{\partial c} = 3b^5h^4 (-1)(5h^3+8c^2)^{-2} \cdot 16c$$