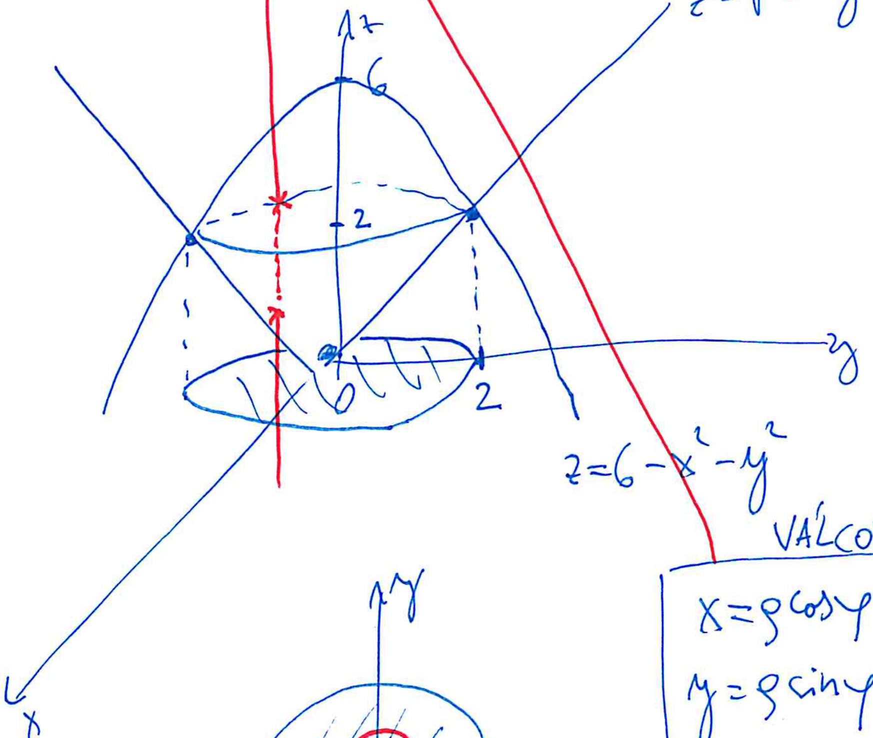
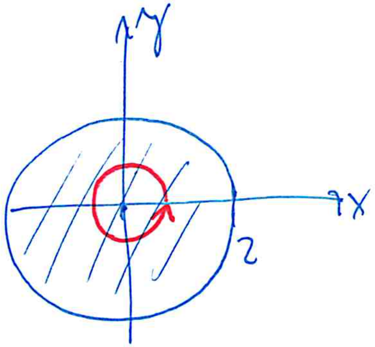
$$\iiint\limits_C \sqrt{x^2+y^2}\,dx\,dy\,dz = \iiint\limits_C \sqrt{\underbrace{\rho^2\cos^2\varphi + \rho^2\sin^2\varphi}_{\rho^2(\cos^2+\sin^2\varphi) = 1}}\;\rho\,d\rho\,d\varphi\,dz = \iiint\limits_C \rho^2\,d\rho\,d\varphi\,dz$$

$$\sqrt{x^2+y^2} \le z \le 6-x^2-y^2$$



$z=\sqrt{x^2+y^2}$

$z=6-x^2-y^2$



$$z=\sqrt{x^2+y^2} \rightarrow z^2 = x^2+y^2 \leftrightarrow 4 = x^2+y^2$$

$$z = 6-(x^2+y^2)$$

průnik: $z = 6 - z^2$

$$z^2+z-6=0$$

$$(z+3)(z-2)=0$$

$z=-3$ $\boxed{z=2}$

$\emptyset$

VÁLCOVÉ: ρ, φ, z

$$x=\rho\cos\varphi$$
$$y=\rho\sin\varphi$$
$$z=z$$

$$0\le\rho\le 2$$
$$0\le\varphi\le 2\pi$$
$$\sqrt{x^2+y^2}\le z\le 6-x^2-y^2$$
$$\rho\le z\le 6-\rho^2$$

$$\iiint_C \varrho^2 \, d\varrho \, d\varphi \, dz = \int_0^{2\pi} \left[\int_0^2 \left(\int_\varrho^{6-\varrho^2} \varrho^2 \, dz \right) d\varrho \right] d\varphi = \int_0^{2\pi} \frac{28}{5} \, d\varphi = \frac{28}{5} \left[\varphi \right]_0^{2\pi} = \boxed{\frac{56}{5}\pi}$$

$0 \le \varrho \le 2$

$0 \le \varphi \le 2\pi$

$\varrho \le z \le 6 - \varrho^2$

1) $$\int_\varrho^{6-\varrho^2} \varrho^2 \, dz = \varrho^2 \left[z \right]_\varrho^{6-\varrho^2} = \varrho^2 \left(6 - \varrho^2 - \varrho \right) = 6\varrho^2 - \varrho^4 - \varrho^3$$

2) $$\int_0^2 \left(6\varrho^2 - \varrho^4 - \varrho^3 \right) d\varrho = \left[2\varrho^3 - \frac{\varrho^5}{5} - \frac{\varrho^4}{4} \right]_0^2 = 16 - \frac{32}{5} - 4 = \frac{60-32}{5} = \underline{\underline{\frac{28}{5}}}$$

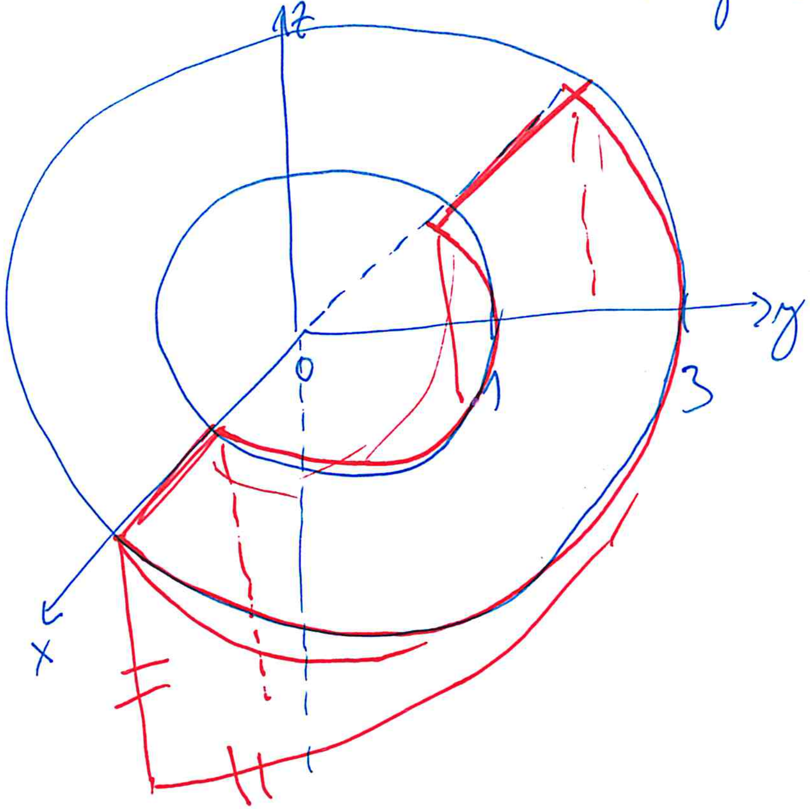
$$\iiint_D z^3\,\underline{dx\,dy\,dz} = \iiint_D r^3\cos^3\vartheta\,\underline{r^2\sin\vartheta\,dr\,d\varphi\,d\vartheta} = \iiint r^5\cos^3\vartheta\sin\vartheta\,dr\,d\varphi\,d\vartheta$$

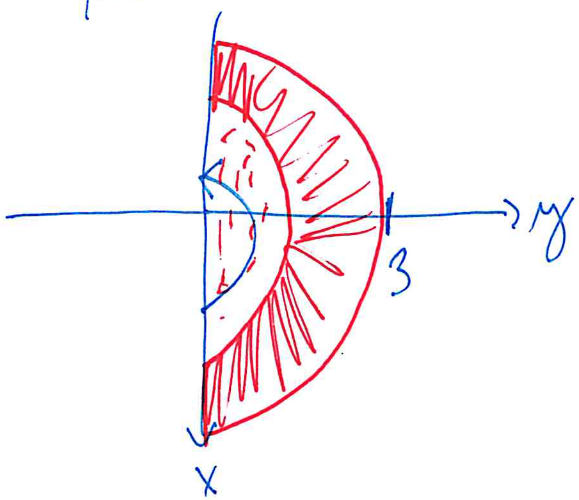
$D:\quad 1 \le x^2+y^2+z^2 \le 9,\quad y \ge 0,\quad z \le 0$

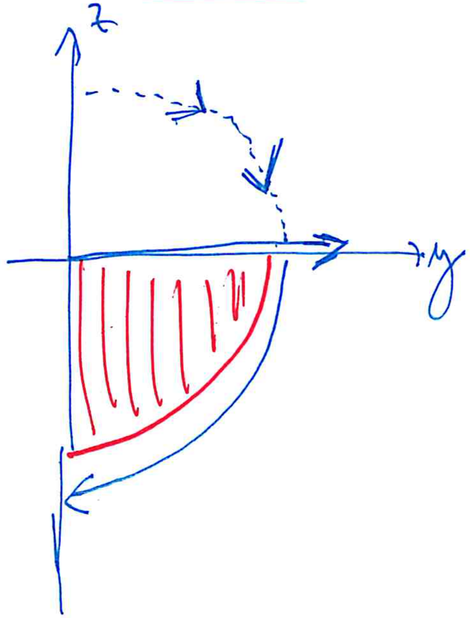
$\longrightarrow y \ge 0$

$$x = r\cos\varphi\sin\vartheta$$
$$y = r\sin\varphi\sin\vartheta$$
$$z = r\cos\vartheta$$
$$J = \underline{r^2\sin\vartheta}$$

$$-\frac{\pi}{2} \le \varphi \le \frac{\pi}{2}$$
$$1 \le r \le 3$$
$$\frac{\pi}{2} \le \vartheta \le \pi$$







$$\iiint r^5 \cos^3\nu \sin\nu \, dr \, d\varphi \, d\nu = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left[\int_{\frac{\pi}{2}}^{\pi} \left(\int_1^3 r^5 \cos^3\nu \sin\nu \, dr \right) d\nu \right] d\varphi =$$

$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} -\frac{91}{\cancel{4}3} \, d\varphi = -\frac{91}{\cancel{4}3} \left[\varphi \right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} = -\frac{91}{3}\pi$$

$$-\frac{\pi}{2} \le \varphi \le \frac{\pi}{2}$$

$$1 \le r \le 3$$

$$\frac{\pi}{2} \le \nu \le \pi$$

1) $$\int_1^3 r^5 \cos^3\nu \sin\nu \, dr = \cos^3\nu \sin\nu \left[\frac{r^6}{6} \right]_1^3 = \left(\frac{729}{6} - \frac{1}{6} \right) \cos^3\nu \sin\nu$$

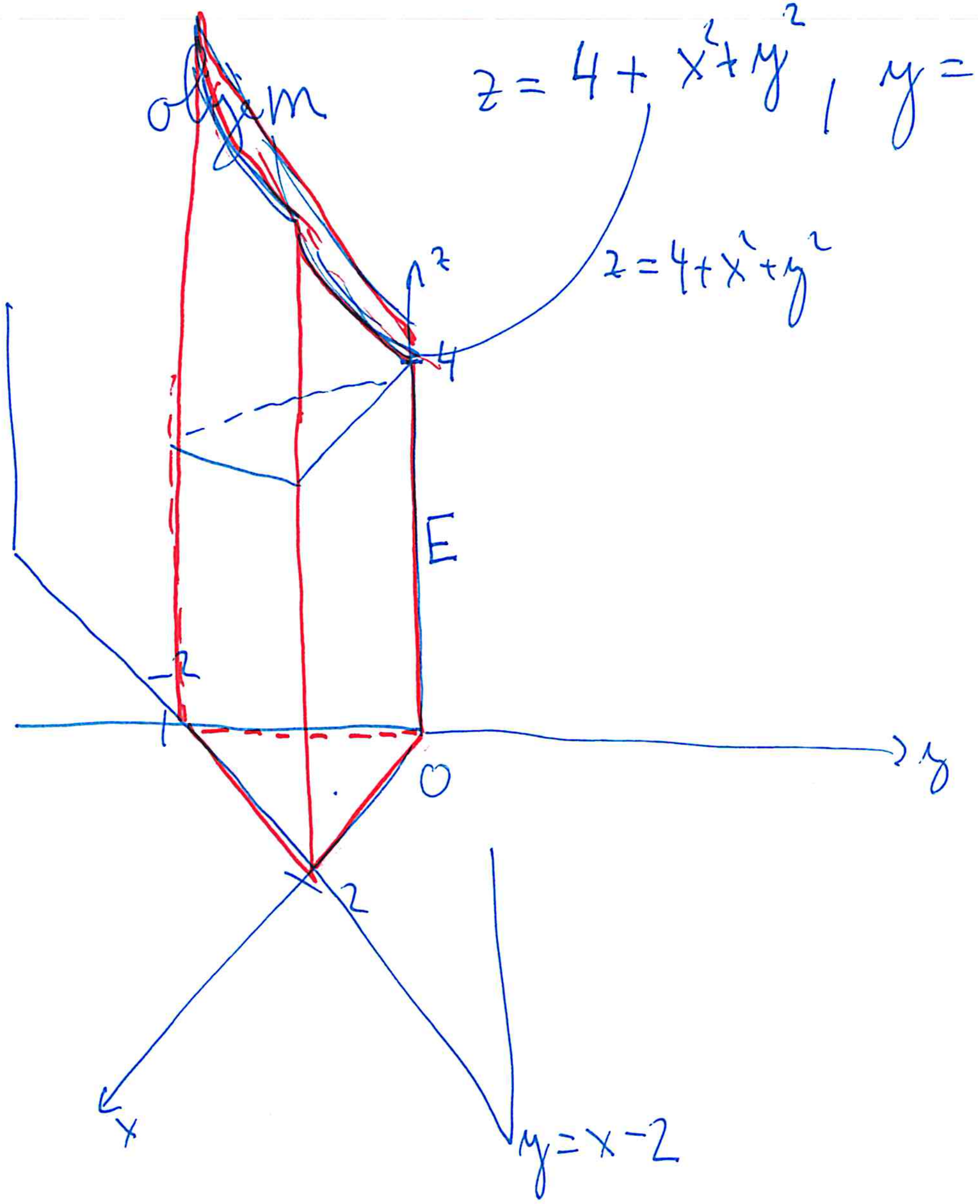
$$= \frac{364}{3} \cos^3\nu \sin\nu$$

2) $$\int_{\frac{\pi}{2}}^{\pi} \frac{364}{3} \cos^3\nu \sin\nu \, d\nu = -\frac{364}{3} \int_0^{-1} t^3 \, dt = \frac{364}{3} \left[\frac{t^4}{4} \right]_{-1}^{0} = \frac{364}{3} \left(0 - \frac{1}{4} \right) = -\frac{91}{\cancel{4}3}$$

$t = \cos\nu$

$dt = -\sin\nu \, d\nu$

objem $z = 4 + x^2 + y^2$, $y = x - 2$, $x = 0$, $y = 0$, $z = 0$

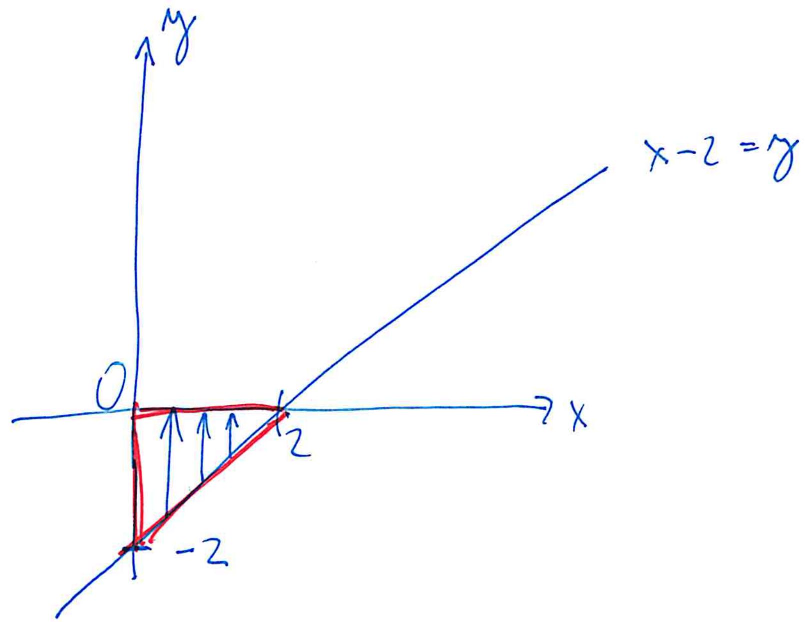


$$V = \iiint_E 1 \, dx\, dy\, dz$$

$$0 \le x \le 2$$

$$x - 2 \le y \le 0$$

$$0 \le z \le 4 + x^2 + y^2$$



$$V = \iiint_E 1\,dx\,dy\,dz = \int_0^2 \left[\int_{x-2}^{0} \left(\int_0^{4+x^2+y^2} 1\,dz \right) dy \right] dx = -\int_0^2 \left(\frac{4}{3}x^3 - 4x^2 + 8x - \frac{32}{3} \right) dx =$$

$$= -\left[\frac{1}{3}x^4 - \frac{4}{3}x^3 + 4x^2 - \frac{32}{3}x \right]_0^2 = \boxed{\frac{32}{3} = V}$$

$0 \leq x \leq 2$

$x-2 \leq y \leq 0$

$0 \leq z \leq 4+x^2+y^2$

1) $$\int_0^{4+x^2+y^2} 1\,dz = \left[z \right]_0^{4+x^2+y^2} = 4+x^2+y^2$$

2) $$\int_{x-2}^{0} (4+x^2+y^2)\,dy = \left[4y + x^2 y + \frac{y^3}{3} \right]_{x-2}^{0} =$$

$$= -\left(4(x-2) + x^2(x-2) + \frac{1}{3}(x-2)^3 \right) = -\left(4x - 8 + x^3 - 2x^2 + \frac{1}{3}\left(x^3 - 6x^2 + 12x - 8 \right) \right) =$$

$$= -\left(\frac{4}{3}x^3 - 4x^2 + 8x - \frac{32}{3} \right)$$