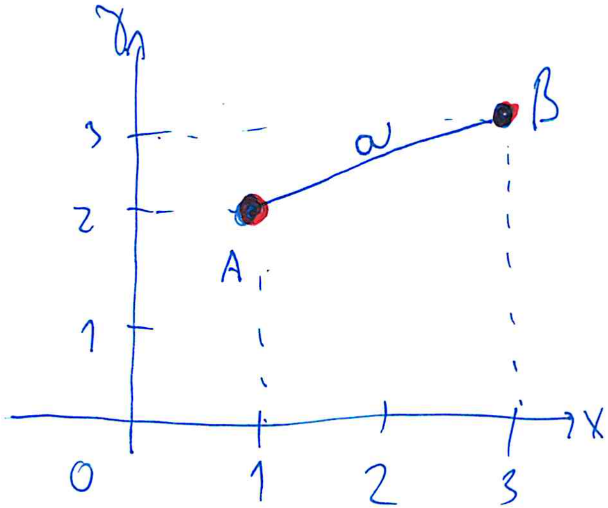
$$\int_a 3x^2y\,ds = 3\sqrt{5}\int_0^1 (1+2t)^2(2+t)\,dt = 3\sqrt{5}\int_0^1 (1+4t+4t^2)(2+t)\,dt =$$

$$= 3\sqrt{5}\int_0^1 (2+t+8t+4t^2+8t^2+4t^3)\,dt =$$

$$= 3\sqrt{5}\int_0^1 (2+9t+12t^2+4t^3)\,dt = *$$

a: úsečka $A = [1,2]$, $B = [3,3]$



$x = 1 + (3-1)t = 1+2t \qquad x' = 2$

$y = 2 + (3-2)t = 2+t \qquad y' = 1$

$X = A + (B-A)\cdot t$

$t \in \langle 0,1 \rangle$

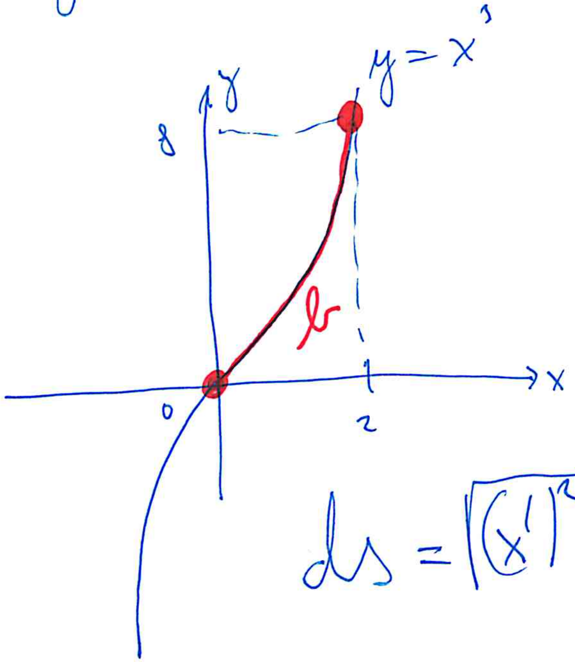
$$ds = \sqrt{(x')^2+(y')^2}\,dt = \sqrt{2^2+1^2}\,dt = \sqrt{5}\,dt$$

$$*) \quad = 3\sqrt{5}\left[2t + \frac{9}{2}t^2 + 4t^3 + t^4\right]_0^1 = 3\sqrt{5}\left(2+\frac{9}{2}+4+1\right) = \frac{69}{2}\sqrt{5}$$

$$\int_k y\,ds = \int_0^2 t^3\sqrt{1+9t^4}\,dt = \frac{1}{36}\int_1^{145}\sqrt{u}\,du = \frac{1}{36}\left[\frac{u^{\frac{3}{2}}}{\frac{3}{2}}\right]_1^{145} = \frac{1}{36}\cdot\frac{2}{3}\left(145^{\frac{3}{2}}-1\right)$$

$$u = 1+9t^4$$
$$du = 36t^3\,dt \rightarrow \frac{1}{36}\,du = t^3\,dt$$

$k: y = x^3$ mezi $[0,0]$, $[2,8]$



$x = t \qquad x' = 1$

$y = t^3 \qquad y' = 3t^2$

$t \in \langle 0, 2\rangle$

$$ds = \sqrt{(x')^2 + (y')^2}\,dt = \sqrt{1+9t^4}\,dt$$

$$\int_C (x^2+y^2+z^2)\,ds = \int_0^{2\pi} (a^2\cos^2 t + a^2\sin^2 t + b^2 t^2)\sqrt{a^2+b^2}\,dt =$$

$$= \sqrt{a^2+b^2}\int_0^{2\pi} (a^2 + b^2 t^2)\,dt = \sqrt{a^2+b^2}\left[a^2 t + b^2\frac{t^3}{3}\right]_0^{2\pi} =$$

$$= \sqrt{a^2+b^2}\left(2\pi a^2 + \frac{b^2}{3}8\pi^3\right)$$

$C:$

$x = a\cos t \qquad x' = -a\sin t$

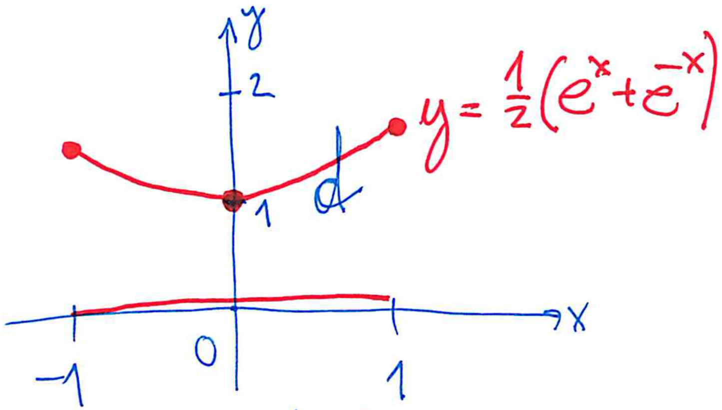
$y = a\sin t \qquad y' = a\cos t$

$z = bt \qquad z' = b$

$t \in \langle 0, 2\pi \rangle$

$$ds = \sqrt{(x')^2 + (y')^2 + (z')^2}\,dt = \sqrt{a^2\sin^2 t + a^2\cos^2 t + b^2}\,dt = \sqrt{a^2+b^2}\,dt$$

Pro $x \in \langle -1, 1 \rangle$ spočítejte délku řetězovky $y = \frac{1}{2}(e^{x} + e^{-x})$



$$l = \int_{d} 1\,ds = \frac{1}{2}\int_{-1}^{1} (e^{t} + e^{-t})\,dt =$$

$$= \frac{1}{2}\left[e^{t} - e^{-t}\right]_{-1}^{1} = \frac{1}{2}\left[e - \frac{1}{e} - \left(\frac{1}{e} - e\right)\right] =$$

$$= \underline{e - \frac{1}{e} \doteq 2{,}35}$$

$t \in \langle -1, 1 \rangle$

$x = t \qquad x' = 1$

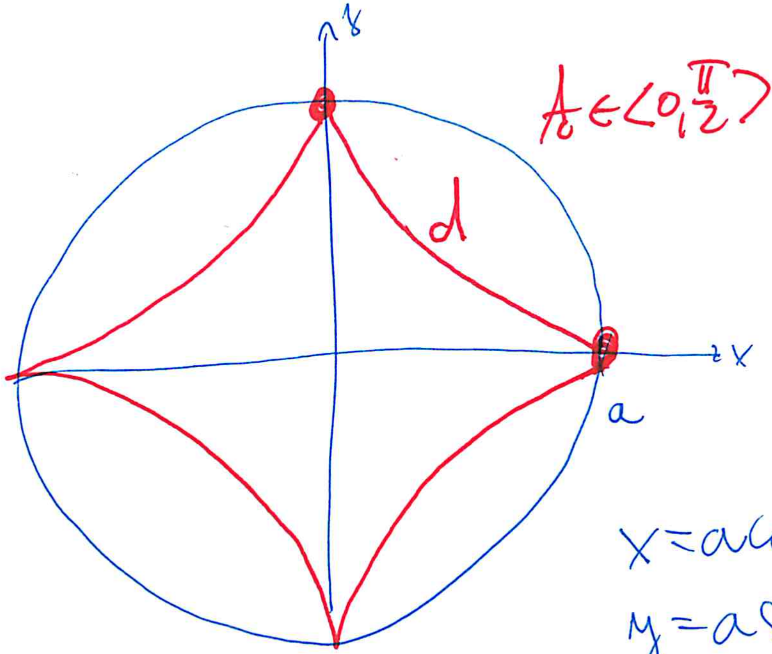
$y = \frac{1}{2}(e^{t} + e^{-t}) \qquad y' = \frac{1}{2}(e^{t} - e^{-t})$

$$ds = \sqrt{(x')^2 + (y')^2}\,dt = \sqrt{1 + \frac{1}{4}\left(e^{2t} - 2\underbrace{e^{t}e^{-t}}_{1} + e^{-2t}\right)}\,dt = \sqrt{\frac{1}{4}e^{2t} + \frac{1}{2} + \frac{1}{4}e^{-2t}}\,dt =$$

$$= \frac{1}{2}\sqrt{e^{2t} + \cancel{\tfrac{1}{4}}\,2\underbrace{e^{t}e^{-t}}_{1} + e^{-2t}}\,dt = \frac{1}{2}\sqrt{(e^{t}+e^{-t})^2}\,dt = \frac{1}{2}(e^{t}+e^{-t})\,dt$$

Spočítejte délku asteroidy $x = a\cos^3 t$, $t \in \langle 0, 2\pi \rangle$

$y = a\sin^3 t$



$$l = 4 \cdot \int_d 1\,ds = 4 \int_0^{\frac{\pi}{2}} \frac{3}{2} a \sin 2t\,dt =$$

$$= 4\,\frac{3}{2}\,a \cdot \frac{1}{2}\left[-\cos 2t\right]_0^{\frac{\pi}{2}} = \frac{\cancel{4}\,3}{\cancel{4}}\,a(1+1) = \underline{\underline{6a}}$$

$x = a\cos^3 t \qquad x' = a\,3\cos^2 t(-\sin t)$

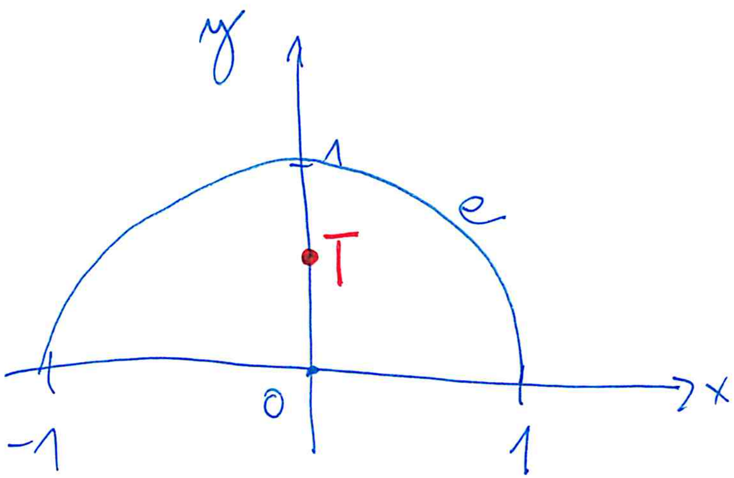
$y = a\sin^3 t \qquad y' = a\,3\sin^2 t(\cos t)$

$$ds = \sqrt{(x')^2 + (y')^2}\,dt = \sqrt{9a^2\cos^4 t\,\sin^2 t + 9a^2\sin^4 t\,\cos^2 t}\;dt =$$

$$= 3a\sqrt{\cos^2 t\,\sin^2 t\,\underbrace{(\cos^2 t + \sin^2 t)}_{1}}\;dt = 3a\cos t\sin t\,dt = \frac{3}{2}a\sin 2t\,dt$$

$\sin 2t = 2\sin t\cos t$

těžiště křivky: $x^2+y^2=1$, $y\geq 0$, $h(x,y)=2-y$



$x_T = 0$

$m = \int_e h(x,y)\,ds$

$y_T = \frac{1}{m}\int_e y\cdot h(x,y)\,ds$

$x=\cos t$, $y=\sin t$, $t\in\langle 0,\pi\rangle$, $x'=-\sin t$, $y'=\cos t$

$ds=\sqrt{(x')^2+(y')^2}\,dt=\sqrt{\sin^2 t+\cos^2 t}\,dt=dt$

$m=\int_e h(x,y)\,ds=\int_e (2-y)\,ds=$

$=\int_0^\pi (2-\sin t)\,dt=$

$=\left[2t+\cos t\right]_0^\pi = 2\pi-1-1=$

$=\underline{\underline{2\pi-2}}$

$$y_T = \frac{1}{m}\int_e y \cdot h \, ds = \frac{1}{m}\int_e y(2-y)\, ds = \frac{1}{2\pi-2}\cdot\frac{8-\pi}{2} = \underline{\underline{\frac{8-\pi}{4\pi-4}}} \doteq 0{,}57$$

$$\int_e y(2-y)\, ds = \int_0^{\pi} \sin t\,(2-\sin t)\, dt = \int_0^{\pi}(2\sin t - \sin^2 t)\, dt =$$

$$\cos 2t = \cos^2 t - \sin^2 t = 1 - 2\sin^2 t \rightarrow$$
$$\sin^2 t = \frac{1-\cos 2t}{2}$$

$$= \int_0^{\pi}\left(2\sin t - \frac{1}{2} + \frac{1}{2}\cos 2t\right) dt =$$

$$= \left[2(-\cos t) - \frac{1}{2}t + \frac{1}{4}\sin 2t\right]_0^{\pi} = 2 - \frac{\pi}{2} - (-2) = 4 - \frac{\pi}{2} = \frac{8-\pi}{2}$$