$$y''' - 3y' + 2y = 8te^{-t}, \quad y(0)=0, \; y'(0)=0, \; y''(0)=1$$

$$\mathcal{L}(8te^{-t}) = \frac{8}{(s+1)^2}, \quad \mathcal{L}(y) = Y, \quad \mathcal{L}(y') = sY, \quad \mathcal{L}(y''') = s^3Y - 1$$

obraz rovnice: $s^3Y - 1 - 3sY + 2Y = \dfrac{8}{(s+1)^2} \quad / + 1$

$$Y(s^3 - 3s + 2) = \frac{8}{(s+1)^2} + 1 \quad / : (s^3 - 3s + 2)$$

$$Y = \frac{8}{(s+1)^2(s^3-3s+2)} + \frac{1}{s^3-3s+2} = \frac{8 + s^2 + 2s + 1}{(s+1)^2(s^3-3s+2)}$$

$$(s^3 - 3s + 2) : (s-1) = s^2 + s - 2 = (s-1)(s+2)$$

$$\begin{array}{l} -(s^3 - s^2) \\ \hline \quad s^2 - 3s + 2 \\ \quad -(s^2 - s) \\ \hline \qquad -2s + 2 \quad -(-2s+2) = 0 \end{array}$$

$$(s^3 - 3s + 2) = (s-1)^2(s+2)$$

$$Y = \frac{s^2+2s+9}{(s+1)^2(s-1)^2(s+2)} = \frac{A}{(s+1)^2} + \frac{B}{s+1} + \frac{C}{(s-1)^2} + \frac{D}{s-1} + \frac{E}{s+2}$$

$$Y = \frac{2}{(s+1)^2} + \frac{1}{(s-1)^2} - \frac{1}{s-1} + \frac{1}{s+2}$$

$$y = 2te^{-t} + te^{t} - e^{t} + e^{-2t}$$

$y_1' - y_2 = 1$ hledáme $y_1 = y_1(t)$ a $y_2 = y_2(t)$

$y_2' + y_1 = t$ $y_1(0) = 1,\ y_2(0) = 0$

$\mathcal{L}(y_1) = Y_1$, $\mathcal{L}(y_1') = sY_1 - 1$, $\mathcal{L}(1) = \frac{1}{s}$

$\mathcal{L}(y_2) = Y_2$, $\mathcal{L}(y_2') = sY_2$, $\mathcal{L}(t) = \frac{1}{s^2}$

obraz soustavy:

$$\boxed{\begin{aligned} sY_1 - 1 - Y_2 &= \frac{1}{s} \\ sY_2 + Y_1 &= \frac{1}{s^2} \end{aligned}}$$

$$sY_1 - Y_2 = \frac{1}{s} + 1$$

$$Y_1 + sY_2 = \frac{1}{s^2} \quad \cdot(-s)$$

$$-Y_2 - s^2 Y_2 = \frac{1}{s} + 1 - \frac{\cancel{s}}{s^{\cancel{2}}} = 1$$

$$Y_2(-1 - s^2) = 1 \quad /:(-1-s^2)$$

$$Y_2 = \frac{1}{-1-s^2} = \frac{-1}{1+s^2} \qquad y_2 = -\sin t$$

$$Y_1 = \frac{1}{s^2} - sY_2 = \frac{1}{s^2} + \frac{s}{1+s^2} \qquad y_1 = t + \cos t$$

$$y_1' + y_2 = e^{-2t} \qquad y_1(0) = y_2(0) = 0$$

$$y_2' - y_1 + 2y_2 = e^{-2t}$$

$\mathcal{L}(y_1) = Y_1$, $\mathcal{L}(y_1') = sY_1$, $\mathcal{L}(y_2) = Y_2$, $\mathcal{L}(y_2') = sY_2$, $\mathcal{L}(e^{-2t}) = \frac{1}{s+2}$

obraz soustavy:

$$sY_1 + Y_2 = \frac{1}{s+2}$$

$$sY_2 - Y_1 + 2Y_2 = \frac{1}{s+2} \quad / \cdot s$$

$$Y_2 + s^2 Y_2 + 2sY_2 = \frac{1}{s+2} + \frac{s}{s+2} = \frac{s+1}{s+2}$$

$$Y_2 \cdot \underbrace{(s^2+2s+1)}_{=(s+1)^2} = \frac{s+1}{s+2} \rightarrow Y_2 = \frac{s+1}{(s+2)(s+1)^2} = \frac{1}{(s+2)(s+1)}$$

$$sY_1 + \frac{1}{(s+2)(s+1)} = \frac{1}{s+2}$$

$$\cancel{s}Y_1 = \frac{1}{s+2} - \frac{1}{(s+2)(s+1)} = \frac{s+1-1}{(s+2)(s+1)} = \frac{\cancel{s}}{(s+2)(s+1)}$$

$$Y_1 = \frac{1}{(s+1)(s+2)} = Y_2$$

$$y_1 = y_2 = \frac{e^{-t} - e^{-2t}}{2-1} = e^{-t} - e^{-2t}$$

$$\frac{1}{(s^2+1)\underbrace{(s^2-1)}_{(s+1)(s-1)}(s^2+4s+10)(s+2)^2} = \frac{Es+F}{s^2+1} + \frac{A}{s+1} + \frac{B}{s-1} +$$

$$+ \frac{Gs+H}{s^2+4s+10} + \frac{D}{(s+2)^2} + \frac{C}{s+2}$$