

$$x^2 \pm a^2, \quad \sqrt{x^2 \pm a^2}, \quad \sqrt{a^2 \pm x^2} \quad (a > 0)$$

$$66. \int \frac{dx}{x^2 + a^2} = \frac{1}{a} \operatorname{arctg} \left(\frac{x}{a} \right) + C \quad 67. \int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln \left| \frac{x-a}{x+a} \right| + C$$

$$68. \int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln |x + \sqrt{x^2 \pm a^2}| + C \quad 69. \int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin \left(\frac{x}{a} \right) + C$$

$$70. \int \frac{x^2}{\sqrt{x^2 \pm a^2}} dx = \frac{x}{2} \sqrt{x^2 \pm a^2} - \frac{\pm a^2}{2} \ln |x + \sqrt{x^2 \pm a^2}| + C$$

$$71. \int \frac{x^2}{\sqrt{a^2 - x^2}} dx = -\frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \arcsin \left(\frac{x}{a} \right) + C$$

$$72. \int \frac{dx}{x\sqrt{a^2 \pm x^2}} = \frac{1}{a} \ln \left| \frac{x}{a + \sqrt{a^2 \pm x^2}} \right| + C$$

$$73. \int \sqrt{x^2 \pm a^2} dx = \frac{x}{2} \sqrt{x^2 \pm a^2} \pm \frac{a^2}{2} \ln |x + \sqrt{x^2 \pm a^2}| + C$$

$$74. \int \sqrt{a^2 - x^2} dx = \frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \arcsin \left(\frac{x}{a} \right) + C$$

$$75. \int x^2 \sqrt{x^2 \pm a^2} dx = \frac{x}{8} (2x^2 \pm a^2) \sqrt{x^2 \pm a^2} - \frac{a^4}{8} \ln |x + \sqrt{x^2 \pm a^2}| + C$$

$$76. \int x^2 \sqrt{a^2 - x^2} dx = \frac{a^4}{8} \arcsin \left(\frac{x}{a} \right) + \frac{x}{8} (2x^2 - a^2) \sqrt{a^2 - x^2} + C$$

Tabulka integrálů

Mocniny

$$1. \int x^n dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1 \quad 2. \int \frac{dx}{x} = \ln |x| + C$$

Exponenciály a logaritmy

$$3. \int e^x dx = e^x + C \quad 4. \int x e^{ax} dx = \frac{x e^{ax}}{a} - \frac{e^{ax}}{a^2} + C$$

$$5. \int x^n e^{ax} dx = \frac{x^n e^{ax}}{a} - \frac{n}{a} \int x^{n-1} e^{ax} dx \quad 6. \int \frac{e^{ax}}{x^n} dx = -\frac{e^{ax}}{(n-1)x^{n-1}} + \frac{a}{n-1} \int \frac{e^{ax}}{x^{n-1}} dx$$

$$7. \int \ln x dx = x \ln x - x + C \quad 8. \int x^n \ln x dx = x^{n+1} \left[\frac{\ln x}{n+1} - \frac{1}{(n+1)^2} \right] + C$$

$$9. \int x^m (\ln x)^n dx = \frac{x^{m+1}}{m+1} (\ln x)^n - \frac{n}{m+1} \int x^m (\ln x)^{n-1} dx$$

$$10. \int \frac{dx}{x \ln x} = \ln |\ln x| + C \quad 11. \int e^{ax} \ln x dx = \frac{e^{ax} \ln x}{a} - \frac{1}{a} \int \frac{e^{ax}}{x} dx$$

Sinus a kosinus

$$12. \int \sin x dx = -\cos x + C \quad 13. \int \cos x dx = \sin x + C$$

$$14. \int \sin^2 x dx = \frac{x}{2} - \frac{\sin 2x}{4} + C \quad 15. \int \cos^2 x dx = \frac{x}{2} + \frac{\sin 2x}{4} + C$$

$$16. \int \sin^n x dx = -\frac{\sin^{n-1} x \cos x}{n} + \frac{n-1}{n} \int \sin^{n-2} x dx$$

$$17. \int \cos^n x dx = \frac{\cos^{n-1} x \sin x}{n} + \frac{n-1}{n} \int \cos^{n-2} x dx$$

$$18. \int x^n \sin x dx = -x^n \cos x + n \int x^{n-1} \cos x dx$$

$$19. \int x^n \cos x dx = x^n \sin x - n \int x^{n-1} \sin x dx$$

$$20. \int \sin mx \cos nx dx = -\frac{\cos[(m+n)x]}{2(m+n)} - \frac{\cos[(m-n)x]}{2(m-n)} + C$$

$$21. \int \sin mx \sin nx \, dx = -\frac{\sin[(m+n)x]}{2(m+n)} + \frac{\sin[(m-n)x]}{2(m-n)} + C$$

$$22. \int \cos mx \cos nx \, dx = \frac{\sin[(m+n)x]}{2(m+n)} + \frac{\sin[(m-n)x]}{2(m-n)} + C$$

$$23. \int \sin(\ln x) \, dx = \frac{x}{2}[\sin(\ln x) - \cos(\ln x)] + C$$

$$24. \int \cos(\ln x) \, dx = \frac{x}{2}[\sin(\ln x) + \cos(\ln x)] + C$$

$$25. \int e^{ax} \sin bx \, dx = \frac{e^{ax}}{a^2 + b^2}(a \sin bx - b \cos bx) + C$$

$$26. \int e^{ax} \cos bx \, dx = \frac{e^{ax}}{a^2 + b^2}(a \cos bx + b \sin bx) + C$$

Tangens a $\frac{1}{\cos x}$

$$27. \int \operatorname{tg} x \, dx = \ln \left| \frac{1}{\cos x} \right| + C$$

$$28. \int \frac{1}{\cos x} \, dx = \ln \left| \frac{1}{\cos x} + \operatorname{tg} x \right| + C$$

$$29. \int \operatorname{tg}^2 x \, dx = \operatorname{tg} x - x + C$$

$$30. \int \frac{1}{\cos^2 x} \, dx = \operatorname{tg} x + C$$

$$31. \int \operatorname{tg}^n x \, dx = \frac{\operatorname{tg}^{n-1} x}{n-1} - \int \operatorname{tg}^{n-2} x \, dx$$

$$32. \int \frac{1}{\cos^n x} \, dx = \frac{\frac{1}{\cos^{n-1} x} \sin x}{n-1} + \frac{n-2}{n-1} \int \frac{1}{\cos^{n-2} x} \, dx$$

$$33. \int \frac{1}{\cos x} \operatorname{tg} x \, dx = \frac{1}{\cos x} + C$$

Kotangens a $\frac{1}{\sin x}$

$$34. \int \operatorname{cotg} x \, dx = \ln |\sin x| + C$$

$$35. \int \frac{1}{\sin x} \, dx = \ln \left| \frac{1}{\sin x} - \operatorname{cotg} x \right| + C$$

$$36. \int \operatorname{cotg}^2 x \, dx = -\operatorname{cotg} x - x + C$$

$$37. \int \frac{1}{\sin^2 x} \, dx = -\operatorname{cotg} x + C$$

$$38. \int \operatorname{cotg}^n x \, dx = -\frac{\operatorname{cotg}^{n-1} x}{n-1} - \int \operatorname{cotg}^{n-2} x \, dx$$

$$39. \int \frac{1}{\sin^n x} \, dx = -\frac{\frac{1}{\sin^{n-1} x} \cos x}{n-1} + \frac{n-2}{n-1} \int \frac{1}{\sin^{n-2} x} \, dx$$

$$40. \int \frac{1}{\sin x} \operatorname{cotg} x \, dx = -\frac{1}{\sin x} + C$$

Inverzní trigonometrické funkce

$$41. \int \arcsin x \, dx = x \arcsin x + \sqrt{1-x^2} + C$$

$$42. \int \arccos x \, dx = x \arccos x - \sqrt{1-x^2} + C$$

$$43. \int \operatorname{arctg} x \, dx = x \operatorname{arctg} x - \frac{1}{2} \ln(1+x^2) + C$$

Hyperbolické funkce

$$44. \int \sinh x \, dx = \cosh x + C$$

$$45. \int \cosh x \, dx = \sinh x + C$$

$$46. \int \operatorname{tgh} x \, dx = \ln(\cosh x) + C$$

$$47. \int \operatorname{cotgh} x \, dx = \ln |\sinh x| + C$$

$$48. \int \frac{1}{\cosh x} \, dx = \arctan(\sinh x) + C$$

$$49. \int \frac{1}{\sinh x} \, dx = \ln \left| \tanh \frac{x}{2} \right| + C$$

$$50. \int \frac{1}{\cosh^2 x} \, dx = \tanh x + C$$

$$51. \int \frac{1}{\sinh^2 x} \, dx = -\operatorname{cotgh} x + C$$

$$52. \int \frac{1}{\cosh x} \operatorname{tanh} x \, dx = -\frac{1}{\cosh x} + C$$

$$53. \int \frac{1}{\sinh x} \operatorname{cotgh} x \, dx = -\frac{1}{\sinh x} + C$$

$$54. \int \sinh^2 x \, dx = \frac{\sinh 2x}{4} - \frac{x}{2} + C$$

$$55. \int \cosh^2 x \, dx = \frac{\sinh 2x}{4} + \frac{x}{2} + C$$

$$56. \int \operatorname{tgh}^2 x \, dx = x - \operatorname{tgh} x + C$$

$$57. \int \operatorname{cotgh}^2 x \, dx = x - \operatorname{cotgh} x + C$$

$$58. \int x \sinh x \, dx = x \cosh x - \sinh x + C$$

$$59. \int x \cosh x \, dx = x \sinh x - \cosh x + C$$

$a + bx, \sqrt{a+bx}$

$$60. \int \frac{x}{a+bx} \, dx = \frac{1}{b^2}(a+bx - a \ln|a+bx|) + C$$

$$61. \int \frac{x^2}{a+bx} \, dx = \frac{1}{b^3} \left[\frac{1}{2}(a+bx)^2 - 2a(a+bx) + a^2 \ln|a+bx| \right] + C$$

$$62. \int \frac{x}{(a+bx)^2} \, dx = \frac{1}{b^2} \left(\frac{a}{a+bx} + \ln|a+bx| \right) + C$$

$$63. \int \frac{x^2}{(a+bx)^2} \, dx = \frac{1}{b^3} \left(a+bx - \frac{a^2}{a+bx} - 2a \ln|a+bx| \right) + C$$

$$64. \int x\sqrt{a+bx} \, dx = \frac{2}{15b^2}(3bx-2a)(a+bx)^{\frac{3}{2}} + C$$

$$65. \int \frac{x}{\sqrt{a+bx}} \, dx = \frac{2}{3b^2}(bx-2a)\sqrt{a+bx} + C$$