1

$$y'' + 9y = 1 + t + \cos 2t \quad , y(0) = 4, y'(0) = -2$$

$$\mathcal{L}(1 + t + \cos 2t) = \frac{1}{s} + \frac{1}{s^2} + \frac{s}{s^2+4}$$

$$\mathcal{L}(y) = Y$$

$$\mathcal{L}(y'') = s^2 Y - s4 + 2$$

$$\left(\mathcal{L}(y') = s\, Y - 4\right)$$

obraz rownie:

$$s^2 Y - s4 + 2 + 9Y = \frac{1}{s} + \frac{1}{s^2} + \frac{s}{s^2+4}$$

$$Y(s^2+9) = \frac{1}{s} + \frac{1}{s^2} + \frac{s}{s^2+4} + 4s - 2 \quad \Big/ : (s^2+9)$$

$$Y = \frac{1}{s(s^2+9)} + \frac{1}{s^2(s^2+9)} + \frac{s}{(s^2+4)(s^2+9)} + \frac{4s}{s^2+9} - \frac{2}{s^2+9}$$

$$y = \frac{1-\cos 3t}{9} + \frac{3t - \sin 3t}{27} + \frac{\cos 2t - \cos 3t}{5} + 4\cos 3t - 2$$

$\frac{1}{3}\sin 3t$

$$y = \frac{1}{9} + \frac{1}{9}t - \frac{19}{27}\sin 3t + \frac{166}{45}\cos 3t + \frac{2}{5}\cos 2t$$

②

$$y'' + 4y' + 4y = t^4 e^{-2t} \qquad \begin{matrix} y(0)=0 \\ y'(0)=0 \end{matrix}$$

$\mathcal{L}(y) = Y$

$\mathcal{L}(y') = sY$ $\qquad \mathcal{L}\left(t^4 e^{-2t}\right) = \dfrac{24}{(s+2)^5}$

$\mathcal{L}(y'') = s^2 Y$ $\qquad \mathcal{L}\left(\dfrac{t^4 e^{-2t}}{4!}\right) = \dfrac{1}{(s+2)^5}$

$n = 5$

$a = +2$

$$\frac{1}{4!} \mathcal{L}\left(t^4 e^{-2t}\right) = \frac{1}{(s+2)^5}$$

obraz rovnice:

$$s^2 Y + 4sY + 4Y = \frac{24}{(s+2)^5}$$

$$Y(s+2)^2 = \frac{24}{(s+2)^5}$$

$$Y = \frac{24}{(s+2)^7}$$

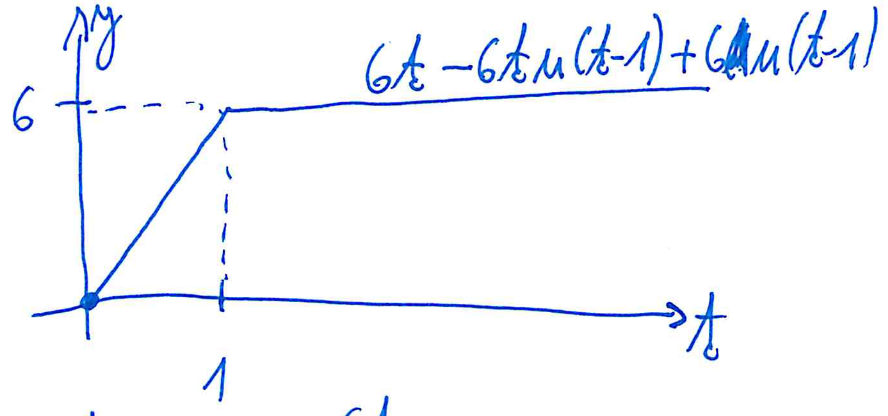
$$y = 24 \frac{t^6 e^{-2t}}{6!} = \underline{\frac{1}{30} t^6 e^{-2t}}$$

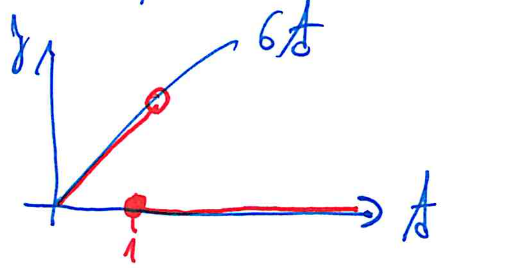
$y'' + 36y = g(t)$, kde $g(t) = 6t \quad t \in \langle 0,1)$

$g(t) = 6 \quad t \geq 1$

$y(0) = 0$

$y'(0) = -6$





$6t - 6t\,u(t-1)$ *

$$g(t) = 6t - 6t\,u(t-1) + 6\,u(t-1)$$

$$\boxed{g(t) = 6t - u(t-1)(6t-6)}$$

pro $t \in \langle 0,1)$ je $u(t-1) = 0$

$$g(t) = 6t$$

pro $t \in \langle 1,+\infty)$ je $u(t-1) = 1$

$$g(t) = 6t - (6t-6) = 6$$

$$g(t) = 6t - 6\,u(t-1)(t-1)$$

$$\mathcal{L}(g(t)) = \frac{6}{s^2} - 6\frac{1}{s^2}e^{-s}$$

$$y'' + 36y = g(t), \quad y(0) = 0, \quad y'(0) = -6$$

$$\mathcal{L}(y) = Y, \quad \mathcal{L}(y'') = s^2Y + 6$$

obraz rownice:

$$s^2Y + 6 + 36Y = \frac{6}{s^2} - \frac{6}{s^2}e^{-s}$$

$$Y(s^2+36) = \frac{6}{s^2} - \frac{6}{s^2}e^{-s} - 6$$

$$Y = \frac{6}{s^2(s^2+36)} - \frac{6}{s^2(s^2+36)}e^{-1s} - \frac{6}{s^2+36}$$

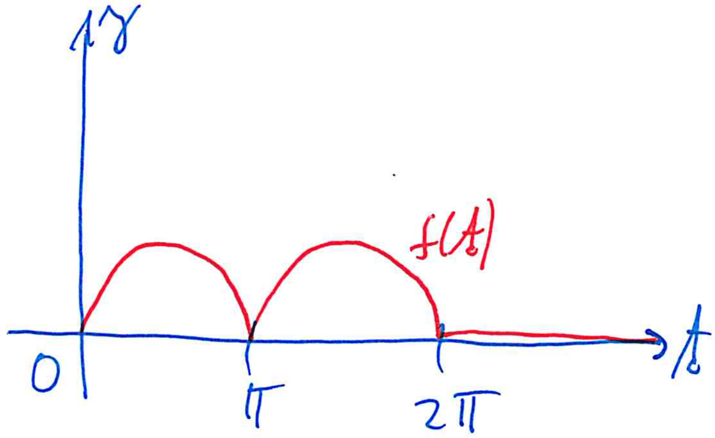
$$y = 6\frac{6t - \sin 6t}{6^3} - 6\frac{6(t-1) - \sin 6(t-1)}{6^3}\cdot u(t-1) - 6\frac{1}{6}\sin 6t$$

$$y'' + 9y = f(t), \quad y(0) = -2, \quad y'(0) = 3$$

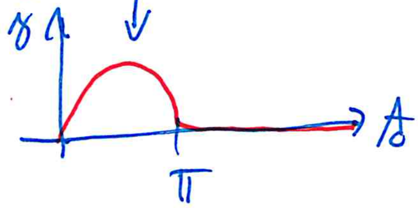
$f(t) = \sin t$ pro $t \in \langle 0, \pi)$

$f(t) = -\sin t$ $\quad t \in \langle \pi, 2\pi)$

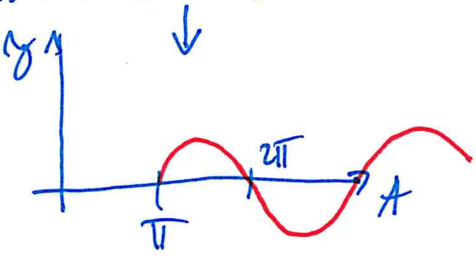
$f(t) = 0$ $\quad t \geq 2\pi$



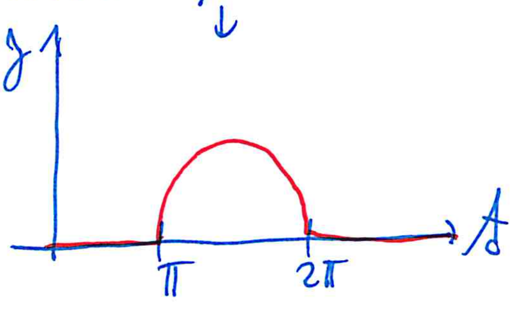
$\sin t - \sin t \cdot u(t - \pi)$



$-\sin t \, u(t - \pi)$



$-\sin t \, u(t - \pi) + \sin t \, u(t - 2\pi)$



$$f(t) = \sin t - 2\sin t \, u(t-\pi) + \sin t \, u(t-2\pi)$$

	$u(t-\pi)$	$u(t-2\pi)$	
$t \in \langle 0, \pi)$	0	0	$f(t) = \sin t$
$t \in \langle \pi, 2\pi)$	1	0	$f(t) = -\sin t$
$t \in \langle 2\pi, +\infty)$	1	1	$f(t) = 0$

$\sin(t-\pi) = -\sin t$

$$f(t) = \sin t + 2\boxed{\sin(t-\pi)^{*} u(t-\pi)} + \sin(t-2\pi)\, u(t-2\pi)$$

$$\mathcal{L}(f(t)) = \frac{1}{s^2+1} + 2\underline{\frac{1}{s^2+1} e^{-\pi s}}^{*} + \frac{1}{s^2+1} e^{-2\pi s}$$

převádíme stejně jako $\sin t$, podle pravidel pro posunutí vyrábíme $e^{-\pi s}$

$$y'' + 9y = f(t), \quad y(0) = -2, \quad y'(0) = 3$$

$$\mathcal{L}(y) = Y, \quad \mathcal{L}(y'') = s^2 Y + 2s - 3$$

obraz rovnice

$$s^2 Y + 2s - 3 + 9Y = \frac{1}{s^2+1} + 2\frac{1}{s^2+1}e^{-\pi s} + \frac{1}{s^2+1}e^{-2\pi s}$$

$$(s^2+9)\cdot Y = \frac{1}{s^2+1} + 2\frac{1}{s^2+1}e^{-\pi s} + \frac{1}{s^2+1}e^{-2\pi s} - 2s + 3$$

$$Y = \frac{1}{(s^2+1)(s^2+9)} + 2\frac{1}{(s^2+1)(s^2+9)}e^{-\pi s} + \frac{1}{(s^2+1)(s^2+9)}e^{-2\pi s} \; \underline{*}$$

$$y = \frac{3\sin t - \sin 3t}{3\cdot 8} + 2\,\frac{3\sin(t-\pi) - \sin 3(t-\pi)}{3\cdot 8}\,u(t-\pi) +$$

$$\underline{*} \quad -\frac{2s}{s^2+9} + \frac{3}{s^2+9}$$

$$-\,2\cos 3t + \cancel{1}\sin 3t + \frac{3\sin(t-2\pi) - \sin 3(t-2\pi)}{3\cdot 8}\cdot u(t-2\pi)$$

$*$

$$y'' + 9y = h(t), \quad y(0) = 0, \; y'(0) = 0$$

DÚ

$h(t) = t \qquad t \in \langle 0, 1)$

$h(t) = 2 - t \qquad t \in \langle 1, 2)$

$h(t) = 0 \qquad t \geq 2$