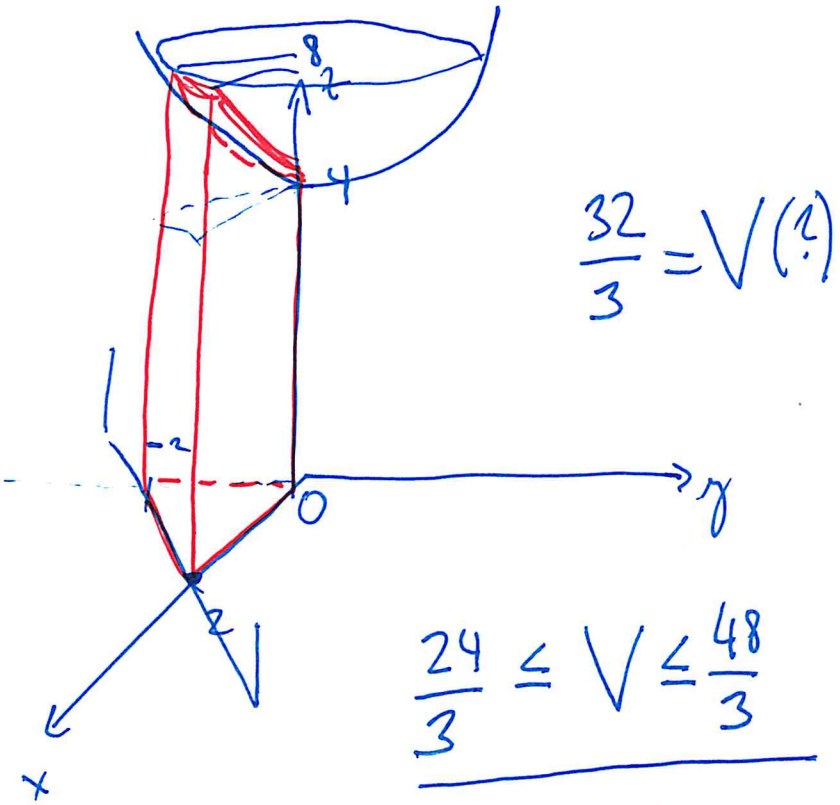
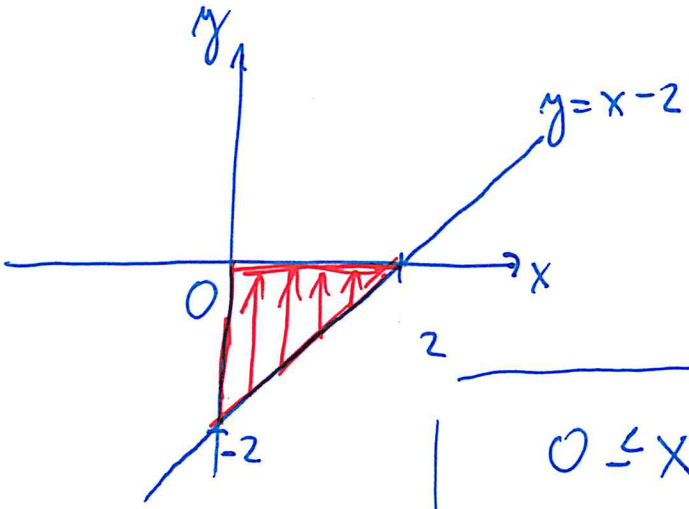
Spočtěte objem tělesa A ohraničeného $z = 4 + x^2 + y^2$, $y = x - 2$

$x = 0$, $y = 0$, $z = 0$



$\frac{32}{3} = V(?)$



$S = 2$

$$\frac{24}{3} \leq V \leq \frac{48}{3}$$

$$V = \iiint_A 1 \, dx\,dy\,dz$$

$$0 \leq x \leq 2$$

$$x - 2 \leq y \leq 0$$

$$0 \leq z \leq 4 + x^2 + y^2$$

$$V = \iiint_A 1 \, dx\,dy\,dz = \int_0^2 \left[\int_{x-2}^{0} \left(\int_0^{4+x^2+y^2} 1 \, dz \right) dy \right] dx =$$

1) $$\int_0^{4+x^2+y^2} 1 \, dz = [z]_0^{4+x^2+y^2} = 4 + x^2 + y^2$$

2) $$\int_{x-2}^{0} (4 + x^2 + y^2)\, dy = \left[4y + x^2 y + \frac{1}{3} y^3 \right]_{x-2}^{0} = -\left(4x - 8 + x^3 - 2x^2 + \frac{1}{3}(x-2)(x^2 - 4x + 4) \right) =$$

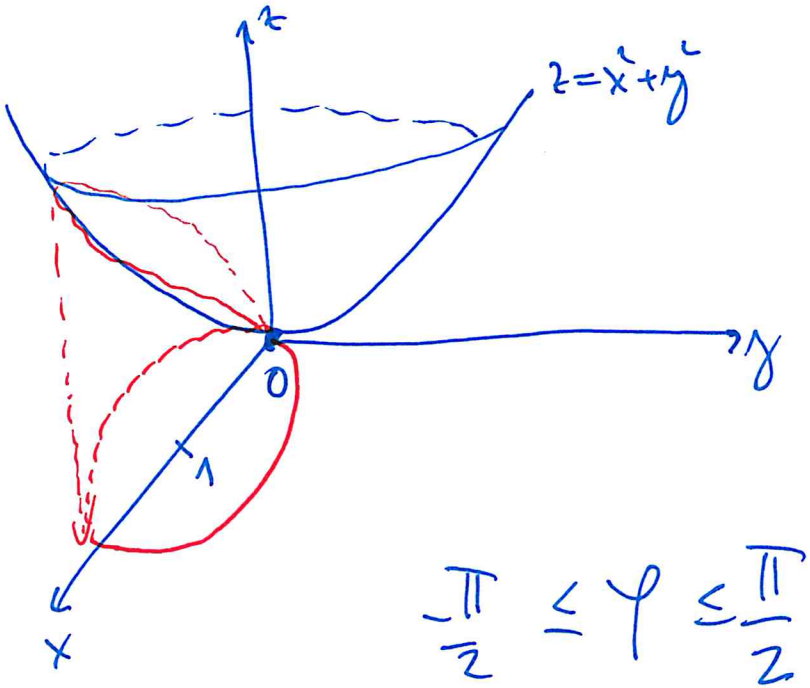
$$= -\left(-2x^2 + x^3 + 4x - 8 + \frac{1}{3}(x^3 - 4x^2 + 4x - 2x^2 + 8x - 8) \right) = -\left(\frac{4}{3}x^3 - 4x^2 + 8x - \frac{32}{3} \right)$$

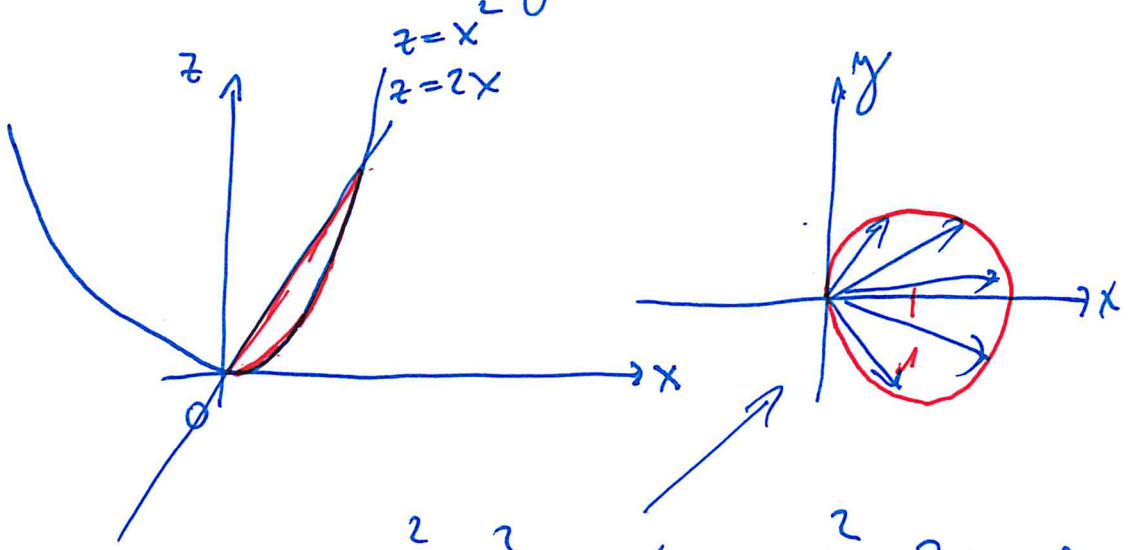
3) $$= \int_0^2 \left(-\frac{4}{3}x^3 + 4x^2 - 8x + \frac{32}{3} \right) dx =$$

$$= \left[-\frac{1}{3}x^4 + 4\frac{x^3}{3} - 4x^2 + \frac{32}{3}x \right]_0^2 = *$$

$$* = -\frac{16}{3} + \frac{32}{3} - 16 + \frac{64}{3} = \frac{-16 + 32 - 48 + 64}{3} = \frac{32}{3}$$

Spočítejte objem tělesa ohraničeného $z = x^2 + y^2$ a $z = 2x$





$x^2 + y^2 = 2x \rightarrow \varrho^2 = 2\varrho\cos\varphi$

$x^2 - 2x + y^2 = 0$

$(x-1)^2 + y^2 = 1$

$\varrho = 2\cos\varphi$

$-\frac{\pi}{2} \leq \varphi \leq \frac{\pi}{2}$

$x = \varrho\cos\varphi$

$y = \varrho\sin\varphi$

$z = z$

$0 \leq \varrho \leq 2\cos\varphi$

$\varrho^2 \leq z \leq 2\varrho\cos\varphi$

$$V = \iiint_B 1 \, dx\,dy\,dz = \iiint_B \varrho \, d\varrho\,d\varphi\,dz$$

$$V = \iiint_B \rho \, d\rho \, d\varphi \, dz = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left[\int_0^{2\cos\varphi} \left(\int_{\rho^2}^{2\rho\cos\varphi} \rho \, dz \right) d\rho \right] d\varphi = \frac{4}{3} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^4\varphi \, d\varphi = * \quad 3)$$

$$-\frac{\pi}{2} \le \varphi \le \frac{\pi}{2}$$

$$0 \le \rho \le 2\cos\varphi$$

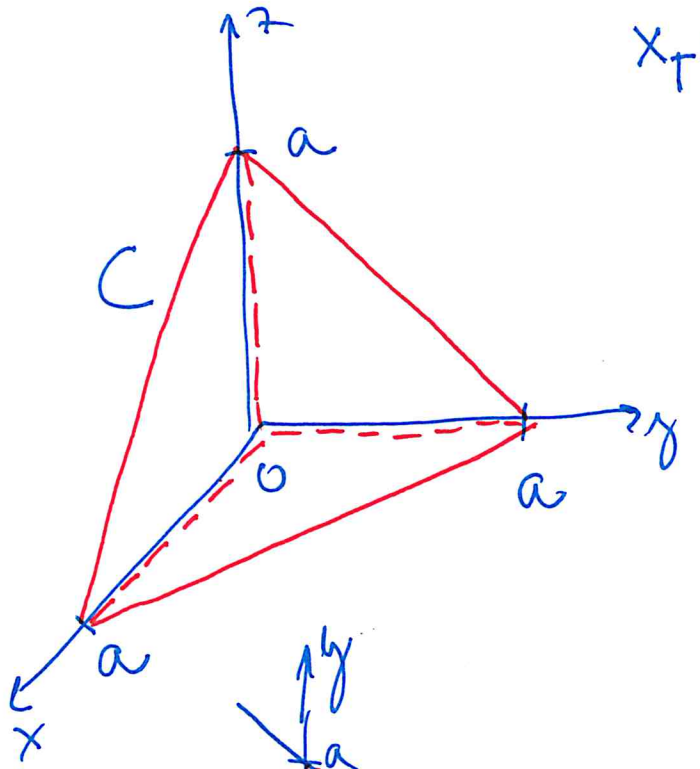
$$\rho^2 \le z \le 2\rho\cos\varphi$$

$$* = \frac{4}{3} \cdot \frac{3\pi}{8} = \frac{\pi}{2}$$

1) $$\int_{\rho^2}^{2\rho\cos\varphi} \rho \, dz = \rho \left[z \right]_{\rho^2}^{2\rho\cos\varphi} = 2\rho^2\cos\varphi - \rho^3$$

2) $$\int_0^{2\cos\varphi} (2\rho^2\cos\varphi - \rho^3) \, d\rho = \left[\frac{2}{3}\rho^3\cos\varphi - \frac{\rho^4}{4} \right]_0^{2\cos\varphi} = \frac{16}{3}\cos^4\varphi - 4\cos^4\varphi = \frac{4}{3}\cos^4\varphi$$

$h(x,y,z)=1 \quad T=[x_T, y_T, z_T] \quad x+y+z=a, \; x=0, y=0, z=0$



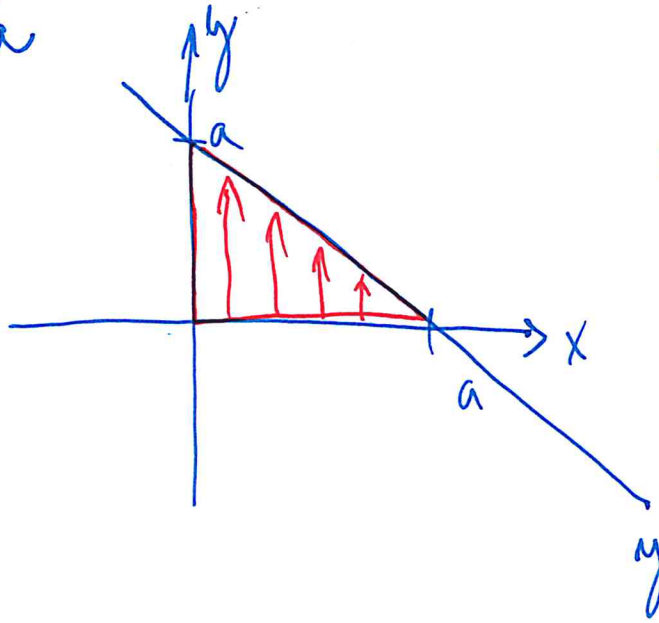
$x_T = y_T = z_T$

$$x_T = \frac{1}{m} \iiint_C x \, dx\,dy\,dz = \frac{6}{a^3} \cdot \frac{1}{24} a^4 = \boxed{\frac{1}{4}a}$$

4)

$$m = h \cdot V = \frac{1}{3} V \cdot S =$$

$$= \frac{1}{3} a \frac{a^2}{2} = \frac{1}{6} a^3$$



$0 \le x \le a$

$0 \le y \le a-x$

$0 \le z \le a-x-y$

$$\iiint_C x\,dx\,dy\,dz = \int_0^a \left[\int_0^{a-x} \left(\int_0^{a-x-y} x\,dz \right) dy \right] dx = \int_0^a \left(\frac{1}{2}a^2x - ax^2 + \frac{1}{2}x^3 \right) dx =$$

3)

$0 \le x \le a$

$0 \le y \le a-x$

$0 \le z \le a-x-y$

$$= \left[\frac{1}{4}a^2x^2 - \frac{1}{3}ax^3 + \frac{1}{8}x^4 \right]_0^a = \frac{1}{4}a^4 - \frac{1}{3}a^4 + \frac{1}{8}a^4 =$$

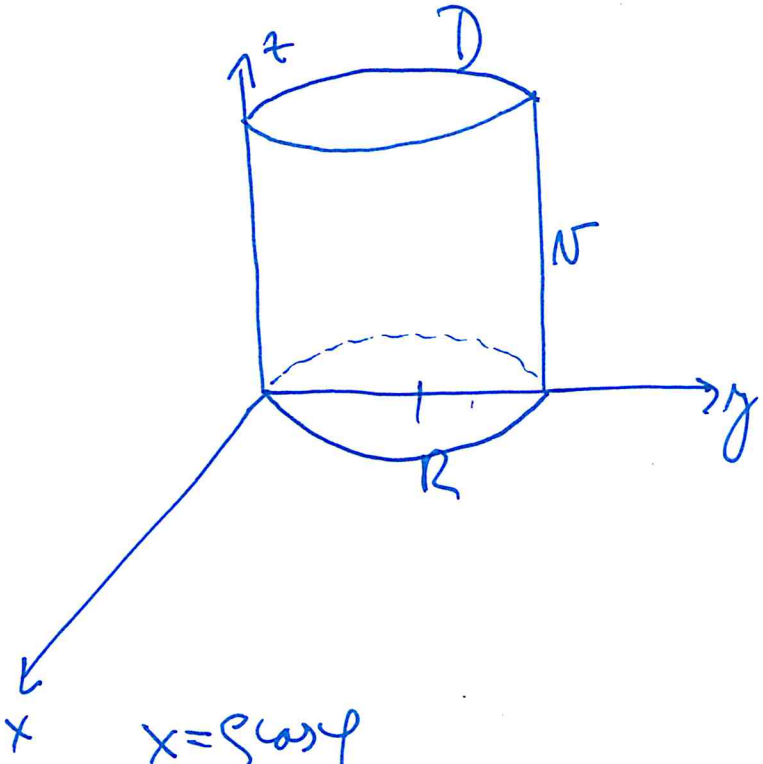
$$= \frac{6-8+3}{24}a^4 = \frac{1}{24}a^4$$

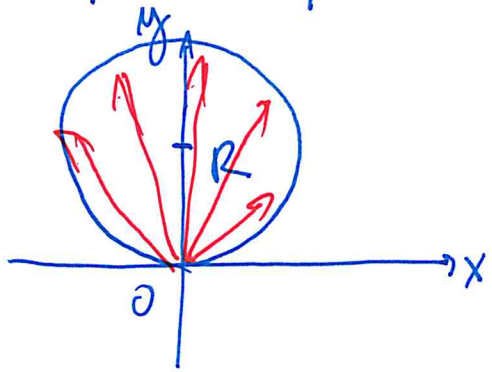
1) $$\int_0^{a-x-y} x\,dz = x\left[z\right]_0^{a-x-y} = ax - x^2 - xy$$

2) $$\int_0^{a-x} (ax - x^2 - xy)\,dy = \left[axy - x^2y - \frac{1}{2}xy^2 \right]_0^{a-x} = ax(a-x) - x^2(a-x) -$$

$$- \frac{1}{2}x(a^2 - 2ax + x^2) = a^2x - ax^2 - ax^2 + x^3 - \frac{1}{2}xa^2 + ax^2 - \frac{1}{2}x^3 = \frac{1}{2}a^2x - ax^2 + \frac{1}{2}x^3$$

Moment setrvačnosti válce R, v, $h(x,y,z)=1$





$x^2+(y-R)^2=R^2$

$x^2+y^2-2yR=0$

$\varrho^2-2\varrho R\sin\varphi=0$

$\boxed{\varrho=2R\sin\varphi}$

$0\le\varphi\le\pi$

$x=\varrho\cos\varphi$ $\quad 0\le\varrho\le 2R\sin\varphi$

$y=\varrho\sin\varphi$ $\quad 0\le z\le v$

$z=z$

$$I_z=\iiint_D (x^2+y^2)\,dx\,dy\,dz=\iiint_D \varrho^3\,d\varrho\,d\varphi\,dz$$

$$\iiint_D \rho^3 \, d\rho \, d\varphi \, dz = \int_0^{\nu} \left[\int_0^{\pi} \left(\int_0^{2R\sin\varphi} \rho^3 \, d\rho \right) d\varphi \right] dz = \int_0^{\nu} \frac{3}{2}\pi R^4 \, dz =$$

$$= \boxed{\frac{3}{2}\pi R^4 \cdot \nu = I_z}$$

$0 \le \varphi \le \pi$

$0 \le \rho \le 2R\sin\varphi$

$0 \le z \le \nu$

$$\int_0^{2R\sin\varphi} \rho^3 \, d\rho = \frac{1}{4}\left[\rho^4\right]_0^{2R\sin\varphi} = 4R^4\sin^4\varphi$$

$$\int_0^{\pi} 4R^4\sin^4\varphi \, d\varphi = 4R^4 \cdot \frac{3\pi}{8} = \frac{3}{2}\pi R^4$$