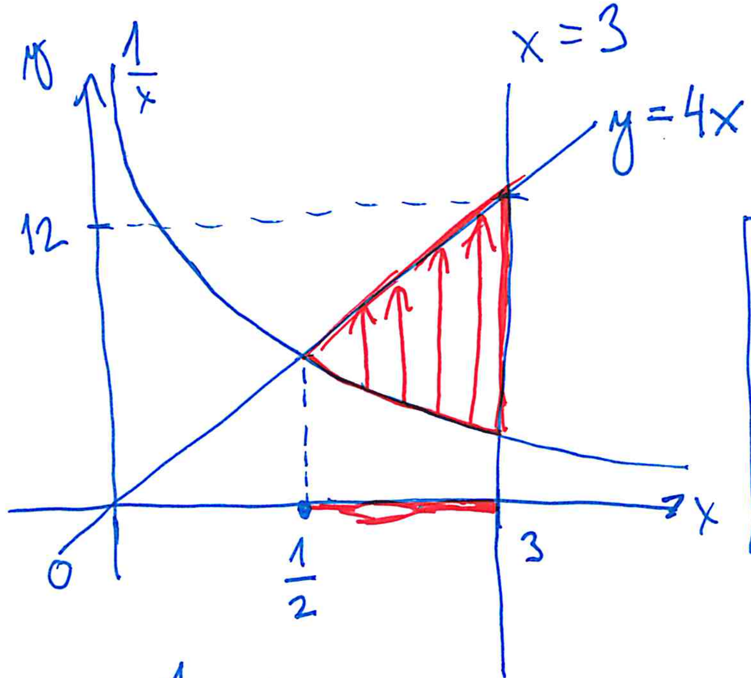
$$\iint\limits_A \frac{x^2}{y^2}\,dx\,dy = \int_{\frac{1}{2}}^{3}\left(\int_{\frac{1}{x}}^{4x}\frac{x^2}{y^2}\,dy\right)dx = \int_{\frac{1}{2}}^{3}\left(x^3-\frac{1}{4}x\right)dx = \left[\frac{x^4}{4}-\frac{x^2}{8}\right]_{\frac{1}{2}}^{3} = *$$

A: obr: $y=\frac{1}{x}$, $y=4x$, $x=3$

$$\int_{\frac{1}{x}}^{4x}\frac{x^2}{y^2}\,dy = x^2\left[\frac{y^{-1}}{-1}\right]_{\frac{1}{x}}^{4x} = -x^2\left[\frac{1}{y}\right]_{\frac{1}{x}}^{4x} =$$



$$\frac{1}{2}\le x\le 3$$
$$\frac{1}{x}\le y\le 4x$$

$$= -x^2\left(\frac{1}{4x}-x\right) =$$

$$= x^3-\frac{1}{4}x$$

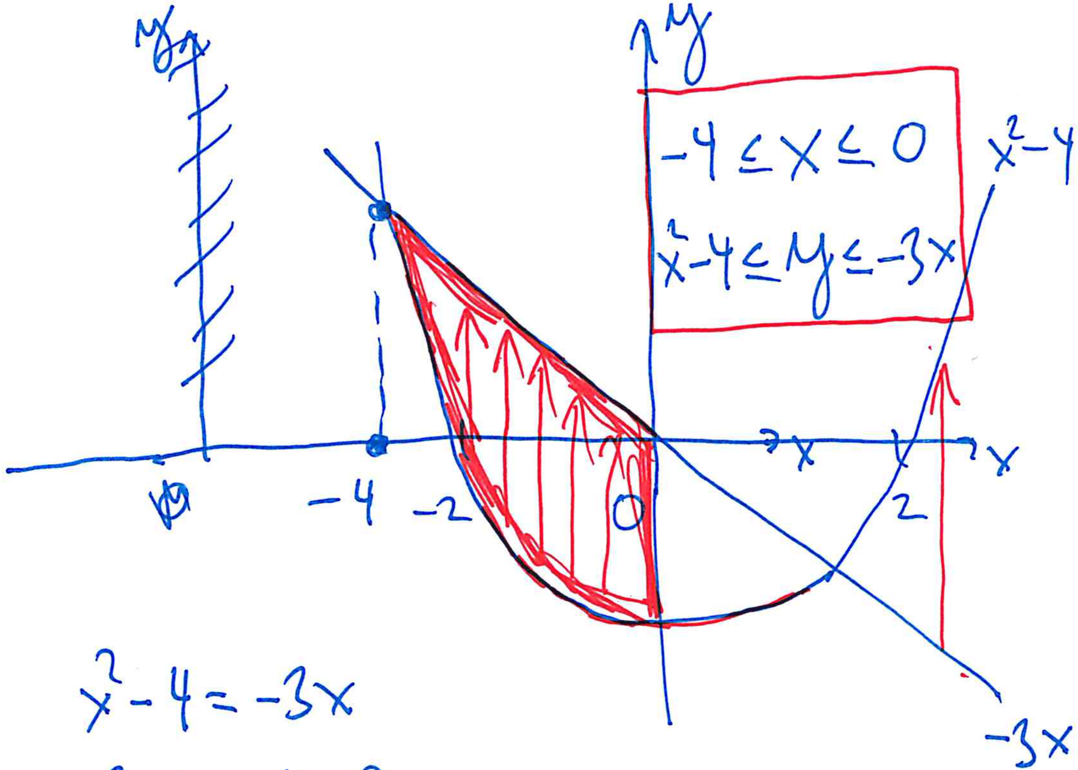
$$\frac{1}{x}=4x$$

$$\frac{1}{4}=x^2$$

$$* = \frac{81}{4}-\frac{9}{8}-\left(\frac{1}{64}-\frac{1}{32}\right)=\frac{1225}{64}$$

$$\iint_B (1+x)\cdot y\,dx\,dy = \int_{-4}^{0}\left(\int_{x^2-4}^{-3x}(1+x)\cdot y\,dy\right)dx = \frac{1}{2}\int_{-4}^{0}\left(17x^2-x^4-16+17x^3-x^5-16x\right)dx$$

B je obrazec ohraničený: $y = x^2-4$, $y=-3x$, pro $x \leq 0$



$$\int_{x^2-4}^{-3x}(1+x)\cdot y\,dy = (1+x)\frac{1}{2}\left[y^2\right]_{x^2-4}^{-3x} =$$

$$= \frac{1}{2}(1+x)\left(9x^2-(x^4-8x^2+16)\right) =$$

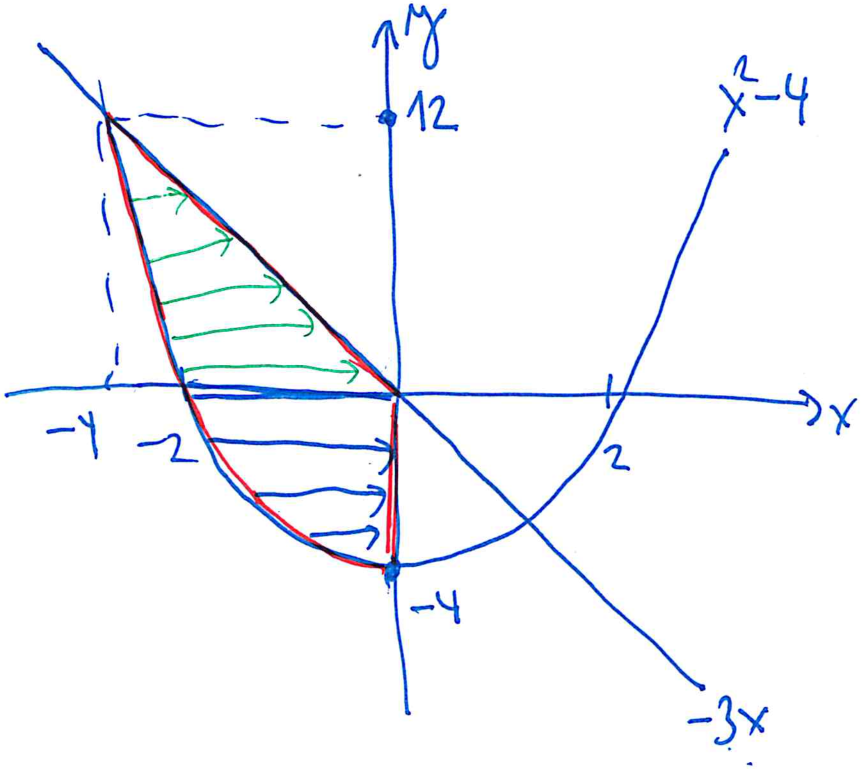
$$= \frac{1}{2}(1+x)\left(17x^2-x^4-16\right) =$$

$$= \frac{1}{2}\left(17x^2-x^4-16+17x^3-x^5-16x\right)$$

$x^2-4=-3x$

$x^2+3x-4=0$

$(x+4)(x-1)=0$



$-4 \le y \le 12$

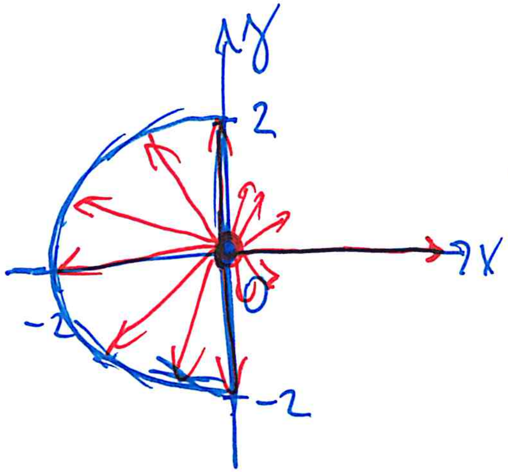
$0 \le y \le 12$ $\quad$ $-4 \le y \le 0$

$x^2-4 \le x \le -3x$ $\quad$ $x^2-4 \le x \le 0$

$$\iint_C \ln(1+x^2+y^2)\,dx\,dy = \iint_C \ln(1+\varrho^2\cos^2\varphi+\varrho^2\sin^2\varphi)\,\varrho\,d\varrho\,d\varphi =$$

$C: x^2+y^2 \le 4$

$x \le 0$



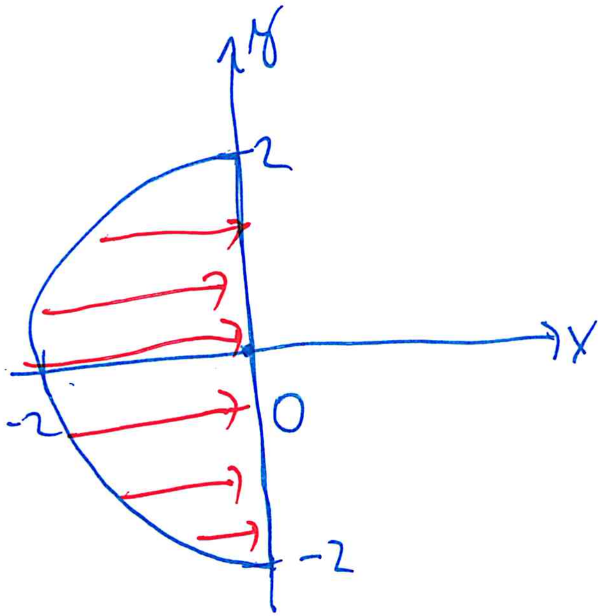
$x=\varrho\cos\varphi$

$y=\varrho\sin\varphi$

$$= \iint_C \ln(1+\varrho^2)\cdot\varrho\,d\varrho\,d\varphi$$

$\frac{\pi}{2} \le \varphi \le \frac{3}{2}\pi$

$0 \le \varrho \le 2$



$x^2+y^2=4$

$x=-\sqrt{4-y^2}$

$-2 \le y \le 2$

$-\sqrt{4-y^2} \le x \le 0$

$$\iint_C \ln(1+\rho^2)\,\rho\, d\rho\, d\varphi = \int_{\frac{\pi}{2}}^{\frac{3}{2}\pi} \left(\int_0^2 \ln(1+\rho^2)\cdot\rho\, d\rho \right) d\varphi = \int_{\frac{\pi}{2}}^{\frac{3}{2}\pi} \frac{1}{2}(5\ln 5 - 4)\, d\varphi =$$

$$= \frac{1}{2}(5\ln 5 - 4)\left[\varphi\right]_{\frac{\pi}{2}}^{\frac{3}{2}\pi} = \boxed{\frac{1}{2}(5\ln 5 - 4)\cdot\pi}$$

$$\frac{\pi}{2} \le \varphi \le \frac{3}{2}\pi$$

$$0 \le \rho \le 2$$

$$\int_0^2 \ln(1+\rho^2)\,\rho\, d\rho = \frac{1}{2}\int_1^5 \ln t\, dt = \frac{1}{2}\left(\left[t\ln t\right]_1^5 - \int_1^5 1\, dt\right) = \frac{1}{2}(5\ln 5 - 4)$$

$$t = 1+\rho^2$$

$$dt = 2\rho\, d\rho$$

$$\frac{1}{2}dt = \rho\, d\rho$$

$$u = \ln t \quad u' = \frac{1}{t}$$

$$v' = 1 \quad v = t$$