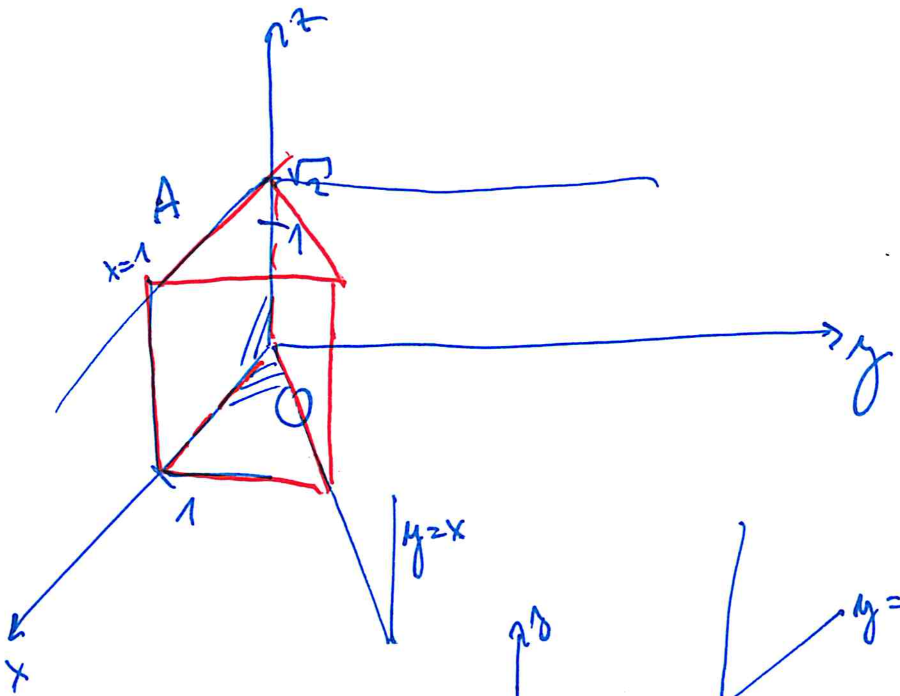
$$\iiint_A (x+y+z)\,dx\,dy\,dz = \int_0^1 \left[\int_0^x \left(\int_0^{\sqrt{2}} (x+y+z)\,dz\right) dy\right] dx = \frac{\sqrt{2}}{2} + \frac{1}{2}$$

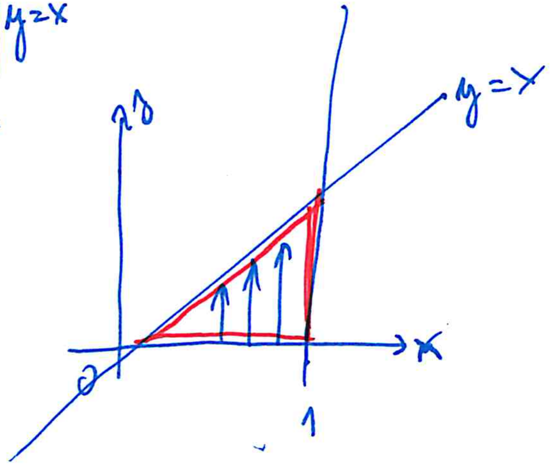
$A:\ x=1,\ y=0,\ y=x,\ z=0,\ z=\sqrt{2}$



$0 \le x \le 1$

$0 \le y \le x$

$0 \le z \le \sqrt{2}$



1) $$\int_0^{\sqrt{2}} (x+y+z)\,dz = \left[xz + yz + \frac{1}{2}z^2\right]_0^{\sqrt{2}} = \sqrt{2}x + \sqrt{2}y + 1$$

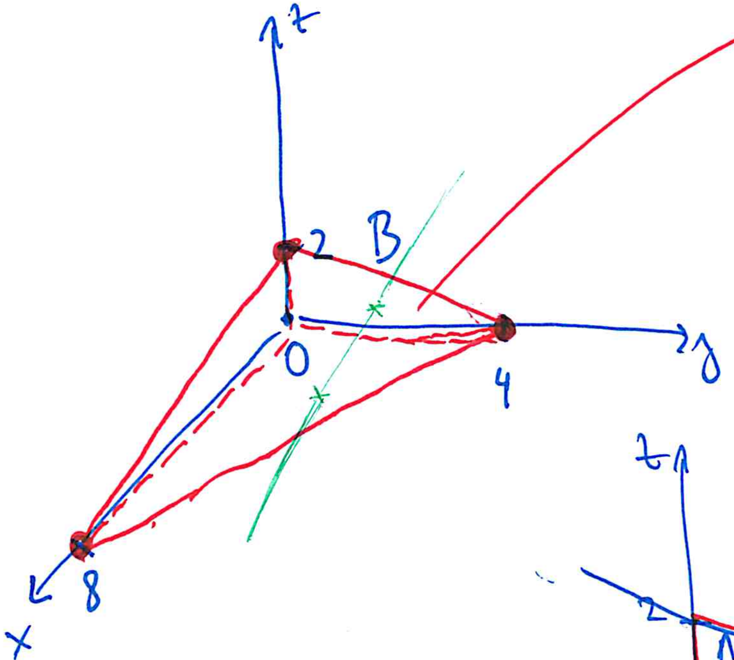
2) $$\int_0^{x} \left(\sqrt{2}x + \sqrt{2}y + 1\right) dy = \left[\sqrt{2}xy + \frac{\sqrt{2}}{2}y^2 + y\right]_0^{x} = \sqrt{2}x^2 + \frac{\sqrt{2}}{2}x^2 + x$$

3) $$\int_0^{1} \left(\frac{3\sqrt{2}}{2}x^2 + x\right) dx = \left[\frac{\sqrt{2}}{2}x^3 + \frac{x^2}{2}\right]_0^{1} = \frac{\sqrt{2}}{2} + \frac{1}{2}$$

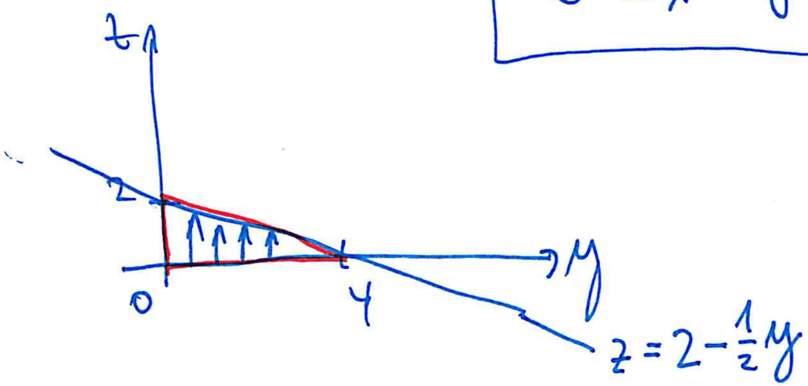
$$\iiint_B y^2\,dx\,dy\,dz = \int_0^4 \left[\int_0^{2-\frac{1}{2}y} \left(\int_0^{8-2y-4z} y^2\,dx \right) dz \right] dy = \underline{\underline{\frac{256}{15}}}$$

$B:\ x=0,\ y=0,\ z=0,\ x+2y+4z=8$

$\hookrightarrow x = 8-2y-4z$



$$\boxed{\begin{aligned} &0 \le y \le 4 \\ &0 \le z \le 2-\tfrac{1}{2}y \\ &0 \le x \le 8-2y-4z \end{aligned}}$$



$$\int_0^{8-2y-4z} y^2\,dx = y^2[x]_0^{8-2y-4z} = y^2(8-2y-4z) = 8y^2-2y^3-4zy^2$$

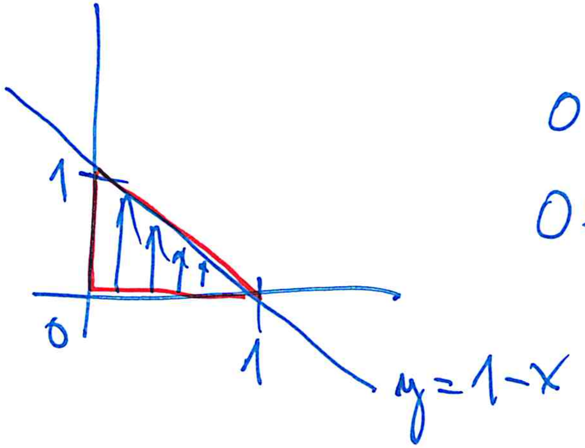
$$\int_0^{2-\frac{1}{2}y}\left(8y^2-2y^3-4zy^2\right)dz = \left[8y^2z-2y^3z-2y^2z^2\right]_0^{2-\frac{1}{2}y} =$$

$$= 8y^2\left(2-\tfrac{1}{2}y\right)-2y^3\left(2-\tfrac{1}{2}y\right)-2y^2\left(4-2y+\tfrac{1}{4}y^2\right) = 16y^2-4y^3-4y^3+y^4-8y^2+4y^3-\tfrac{1}{2}y^4 =$$

$$= 8y^2-4y^3+\tfrac{1}{2}y^4$$

$$\int_0^4\left(8y^2-4y^3+\tfrac{1}{2}y^4\right)dy = \left[8\frac{y^3}{3}-y^4+\frac{y^5}{10}\right]_0^4 = \frac{8}{3}\cdot 64-256+\frac{1024}{10} = \frac{256}{15}$$

$$\iint_F 1\,dx\,dy = \int_0^{1-x}\left(\int_0^1 1\,dx\right)dy = \int_0^{1-x}[x]_0^1\,dy = \int_0^{1-x} 1\,dy =$$



$$0 \le x \le 1$$
$$0 \le y \le 1-x$$

$$= [y]_0^{1-x} = \underline{\underline{1-x}} \text{ ??}$$

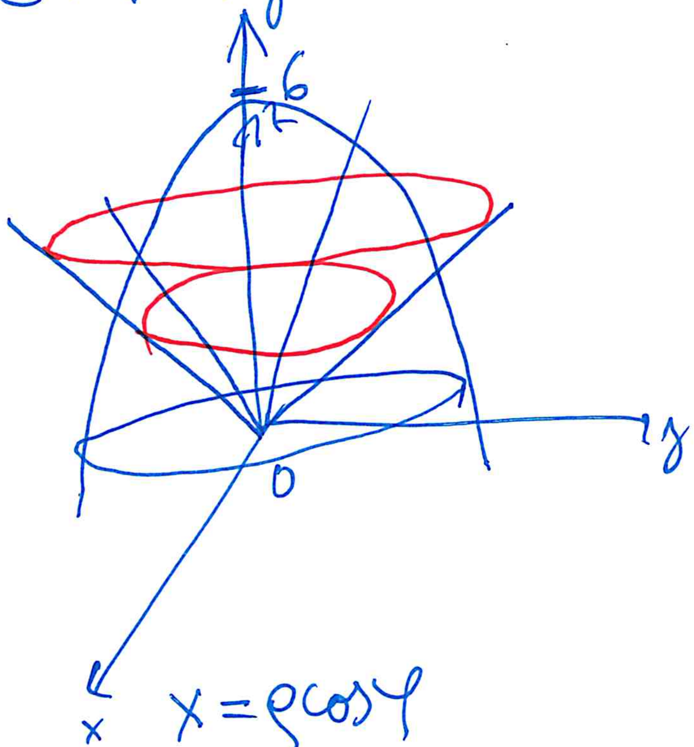
$$\int_0^1\left(\int_0^{1-x} 1\,dy\right)dx = \int_0^1 (1-x)\,dx = \left[x-\frac{x^2}{2}\right]_0^1 = \underline{\underline{\frac{1}{2}}}$$

$$\iiint_C \sqrt{x^2+y^2}\,dx\,dy\,dz$$

$z=\sqrt{x^2+y^2}=\sqrt{\rho^2\cos^2\varphi+\rho^2\sin^2\varphi}=\sqrt{\rho^2}=|\rho|=\rho$

$z=6-x^2-y^2=6-\rho^2\cos^2\varphi-\rho^2\sin^2\varphi=6-\rho^2$

$C: \sqrt{x^2+y^2} \le z \le 6-x^2-y^2$



$z=\sqrt{x^2+y^2}\quad \left(z^2=\underbrace{x^2+y^2}\right)$

$z=6-x^2-y^2=6-(x^2+y^2)$

$\downarrow$

$z=6-z^2$

$z^2+z-6=0$

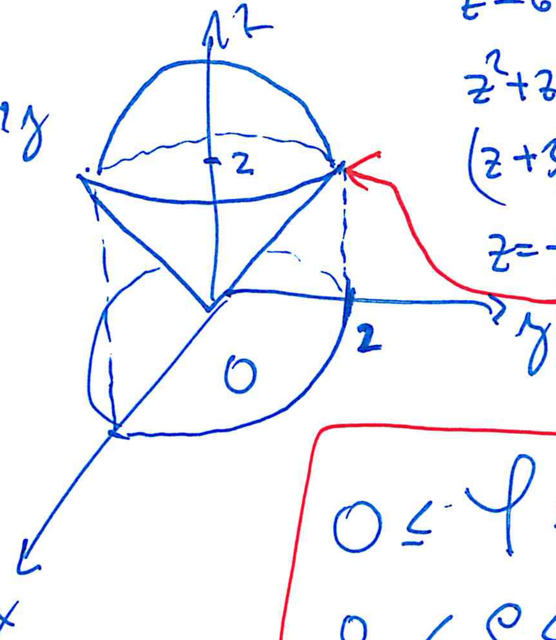
$(z+3)(z-2)=0$

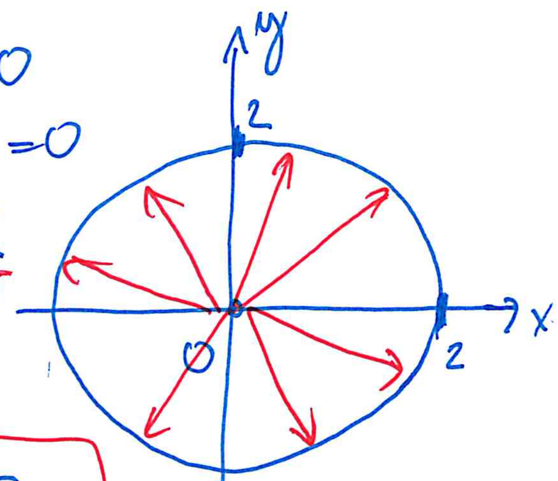
$z=-3$ ✗

$\boxed{z=2}$

$x=0 \to z=\sqrt{y^2}=|y|$

$y=0 \to z=\sqrt{x^2}=|x|$





$x=\rho\cos\varphi$

$y=\rho\sin\varphi$

$z=z$

$$\boxed{\begin{aligned} 0 &\le \varphi \le 2\pi \\ 0 &\le \rho \le 2 \\ \rho &\le z \le 6-\rho^2 \end{aligned}}$$

$$\iiint_C \sqrt{x^2+y^2}\,dx\,dy\,dz = \iiint_C \rho^2\,d\rho\,d\varphi\,dz = \int_0^{2\pi}\left[\int_0^2\left(\int_\rho^{6-\rho^2}\rho^2\,dz\right)d\rho\right]d\varphi = \frac{56}{5}\pi$$

4)

$0 \leq \varphi \leq 2\pi$

$0 \leq \rho \leq 2$

$\rho \leq z \leq 6-\rho^2$

$x = \rho\cos\varphi$

$y = \rho\sin\varphi$

$z = z$

1) $$\int_\rho^{6-\rho^2} \rho^2\,dz = \rho^2\left[z\right]_\rho^{6-\rho^2} = \rho^2(6-\rho^2-\rho) = 6\rho^2-\rho^4-\rho^3$$

2) $$\int_0^2 (6\rho^2-\rho^4-\rho^3) = \left[2\rho^3-\frac{\rho^5}{5}-\frac{\rho^4}{4}\right]_0^2 = 16-\frac{32}{5}-4 = 12-\frac{32}{5} = \frac{28}{5}$$

3) $$\int_0^{2\pi} \frac{28}{5}\,d\varphi = \frac{28}{5}\left[\varphi\right]_0^{2\pi} = \frac{56}{5}\pi$$

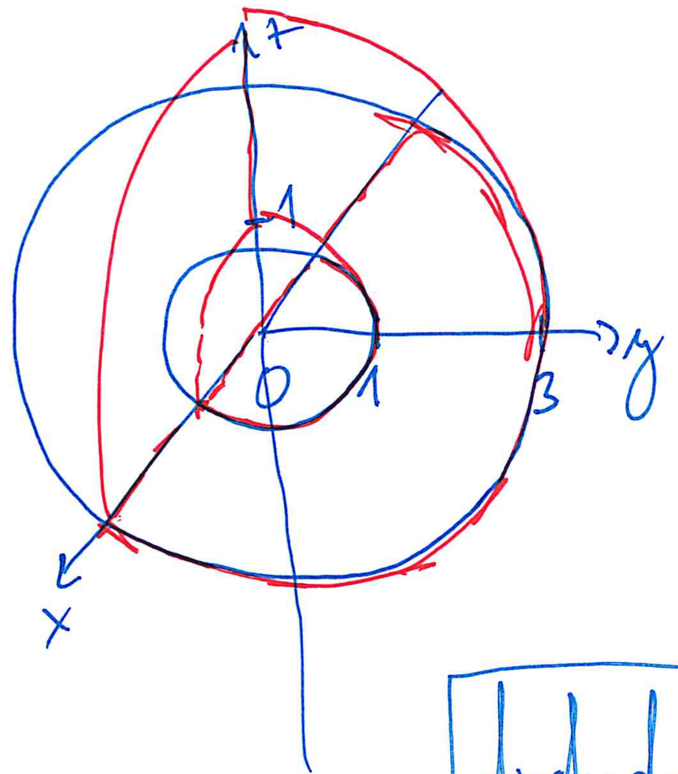
$$\iiint_D z^3\,dx\,dy\,dz = \iiint_D r^5\cos^3\theta\,\sin\theta\,dr\,d\varphi\,d\theta$$

$D:\ 1 \le x^2+y^2+z^2 \le 9,\quad 0 \le y,\quad 0 \le z$

$0 \le \theta \le \frac{\pi}{2}$

$0 \le \varphi \le \pi$

$1 \le r \le 3$

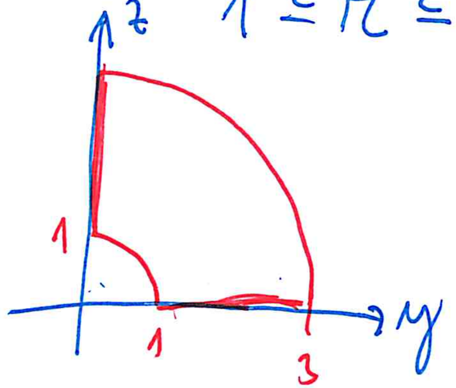


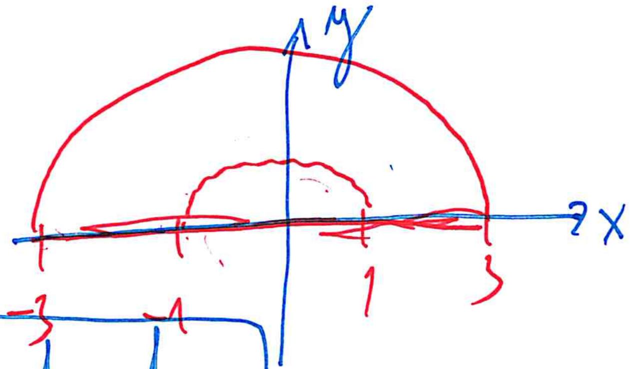
$x = r\cos\varphi\sin\theta$

$y = r\sin\varphi\sin\theta$

$z = r\cos\theta$

$J = r^2\sin\theta$





$$\boxed{dx\,dy\,dz = r^2\sin\theta\,dr\,d\varphi\,d\theta}$$

$$\iiint_D r^5 \cos^3\vartheta \sin\vartheta \, dr\, d\varphi\, d\vartheta = \int_0^{\pi}\left[\int_1^3\left(\int_0^{\frac{\pi}{2}} r^5 \cos^3\vartheta \sin\vartheta \, d\vartheta\right) dr\right] d\varphi = \underline{\underline{\frac{91}{3}\pi}}$$

4)

1) $$\int_0^{\frac{\pi}{2}} r^5 \cos^3\vartheta \sin\vartheta \, d\vartheta = -r^5 \int_1^0 t^3\, dt = r^5 \left[\frac{t^4}{4}\right]_0^1 = \frac{1}{4} r^5$$

$t = \cos\vartheta$

$dt = -\sin\vartheta \, d\vartheta$

2) $$\int_1^3 \frac{1}{4} r^5 \, dr = \left[\frac{1}{4}\frac{r^6}{6}\right]_1^3 = \frac{1}{24}(729-1) = \frac{728}{24} = \frac{91}{3}$$

3) $$\int_0^{\pi} \frac{91}{3} d\varphi = \frac{91}{3}\left[\varphi\right]_0^{\pi} = \underline{\underline{\frac{91}{3}\pi}}$$