1)
$$\int_C (x^2+y^2+z^2)\,ds = \int_0^{2\pi} (a^2\cos^2 t + a^2\sin^2 t + b^2 t^2)\sqrt{a^2+b^2}\,dt = *$$

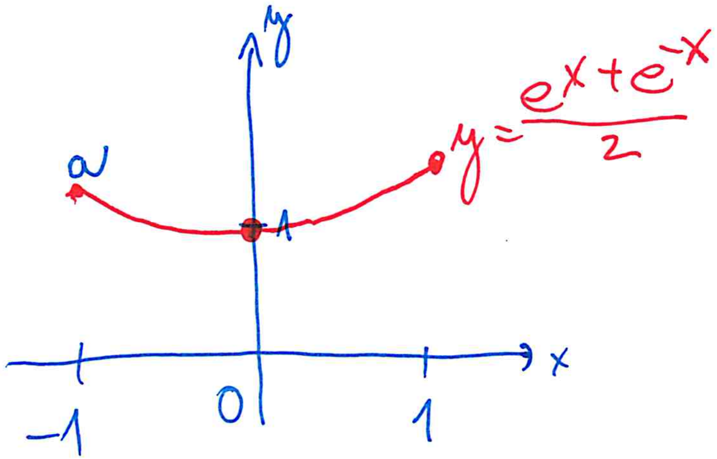
$$C: \quad x = a\cos t, \quad y = a\sin t, \quad z = bt, \quad t \in \langle 0, 2\pi \rangle$$

$$x' = -a\sin t, \quad y' = a\cos t, \quad z' = b$$

$$ds = \sqrt{x'^2+y'^2+z'^2}\,dt = \sqrt{a^2\sin^2 t + a^2\cos^2 t + b^2}\,dt = \sqrt{a^2+b^2}\,dt$$

$$* = \sqrt{a^2+b^2}\int_0^{2\pi}(a^2+b^2t^2)\,dt = \sqrt{a^2+b^2}\left[a^2 t + b^2\frac{t^3}{3}\right]_0^{2\pi} = \underline{\underline{\left(a^2\cdot 2\pi + b^2\frac{8\pi^3}{3}\right)\sqrt{a^2+b^2}}}$$

2) Pro $x \in \langle -1, 1 \rangle$ spočítejte délku řetězovky $y = \dfrac{e^x + e^{-x}}{2}$



$$l = \int_a 1\, ds = \frac{1}{2}\int_{-1}^{1} \left(e^t + e^{-t}\right) dt =$$

$$= \frac{1}{2}\left[e^t - e^{-t}\right]_{-1}^{1} = \frac{1}{2}\left(e - \frac{1}{e} - \left(\frac{1}{e} - e\right)\right) =$$

$$= \underline{\underline{e - \frac{1}{e}}}$$

$x = t \qquad x' = 1$

$y = \dfrac{e^t + e^{-t}}{2} \qquad y' = \dfrac{1}{2}\left(e^t - e^{-t}\right)$

$t \in \langle -1, 1 \rangle$

$$ds = \sqrt{x'^2 + y'^2}\, dt = \sqrt{1 + \frac{1}{4}\left(e^{2t} - 2\underbrace{e^t e^{-t}}_{1} + e^{-2t}\right)}\, dt =$$

$$= \sqrt{\frac{1}{4}e^{2t} + \frac{1}{2} + \frac{1}{4}e^{-2t}}\, dt = \frac{1}{2}\sqrt{e^{2t} + 2e^t e^{-t} + e^{-2t}}\, dt = \frac{1}{2}\sqrt{\left(e^t + e^{-t}\right)^2}\, dt$$

$$= \frac{1}{2}\left(e^t + e^{-t}\right) dt$$

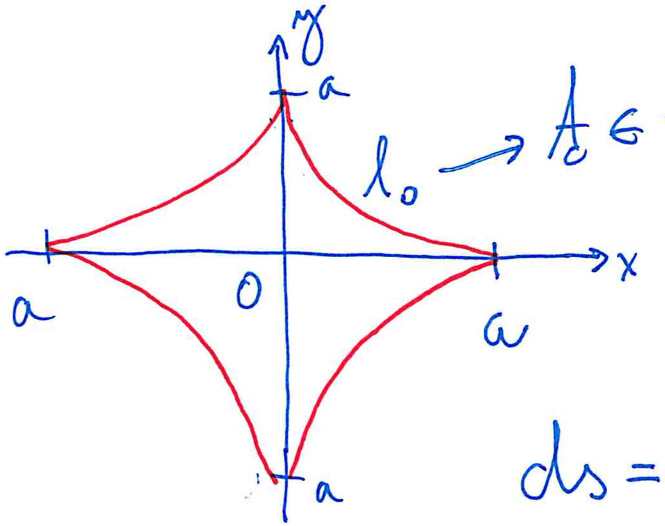
3) Délka asteroidy $x = a\cos^3 t$

$y = a\sin^3 t$

$t \in \langle 0, 2\pi \rangle$

$$l_0 = \int 1\,ds = \int_0^{\frac{\pi}{2}} \frac{3}{2} a \sin 2t\, dt =$$

$$= \frac{3}{2} a \left[-\cos 2t\right]_0^{\frac{\pi}{2}} \cdot \frac{1}{2} = \frac{3}{4} a (1-(-1)) = \frac{3}{2} a$$



$l_0 \rightarrow t \in \langle 0, \frac{\pi}{2} \rangle$

$$\boxed{l_{celé} = 4 l_0 = 6a}$$

$$ds = \sqrt{9a^2 \cos^4 t \sin^2 t + 9a^2 \sin^4 t \cos^2 t}\, dt =$$

$$= 3a \sqrt{\cos^2 t \sin^2 t \underbrace{(\cos^2 t + \sin^2 t)}_{1}}\, dt =$$

$x' = -a\, 3\cos^2 t \cdot \sin t$

$y' = a\, 3\sin^2 t \cos t$

$$= 3a \cos t \sin t\, dt = \frac{3}{2} a \sin 2t\, dt$$

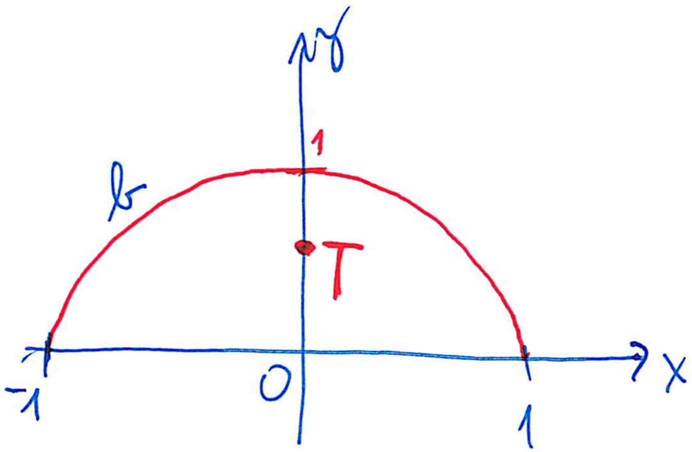
$\sqrt{\cos^2 t} = |\cos t|$

$t \in \langle 0, \frac{\pi}{2} \rangle$

$\sin 2t = 2 \sin t \cos t$

4) Nalezněte souřadnice těžiště $c: x^2+y^2=1$ pro $y \geq 0$

$h(x,y)=2-y$



$x_T = 0$

$\cancel{m = h \cdot l}$

$$m = \int_b h(x,y)\,ds = \int_b (2-y)\,ds =$$

$$= \int_0^{\pi} (2-\sin t)\,dt = \left[2t+\cos t\right]_0^{\pi} = 2\pi - 1 - (1) =$$

$$= \underline{2\pi - 2}$$

$x = \cos t \qquad x' = -\sin t$

$y = \sin t \qquad y' = \cos t$

$t \in \langle 0, \pi \rangle$

$ds = \sqrt{\sin^2 t + \cos^2 t}\,dt = dt$

$x = \cos t$

$y = \sin t$

$t \in \langle 0, \pi \rangle$

$ds = dt$

$m = 2\pi - 2$

$$y_T = \frac{1}{m}\int_b y \cdot h \, ds = \frac{1}{2\pi - 2}\int_b y \cdot (2 - y)\, ds = \frac{4 - \frac{\pi}{2}}{2\pi - 2} =$$

$$= \frac{8 - \pi}{4\pi - 4} \doteq 0{,}57$$

$$\int_b (2y - y^2)\, ds = \int_0^{\pi} (2\sin t - \sin^2 t)\, dt = \int_0^{\pi} \left(2\sin t - \frac{1}{2} + \frac{1}{2}\cos 2t\right) dt =$$

$$= \left[-2\cos t - \frac{1}{2}t + \frac{1}{4}\sin 2t\right]_0^{\pi} = \left(2 - \frac{\pi}{2} - (-2)\right) = 4 - \frac{\pi}{2}$$

$\cancel{\cos 2t}$

$$\cos 2t = \cos^2 t - \sin^2 t = 1 - 2\sin^2 t \rightarrow \sin^2 t = \frac{1 - \cos 2t}{2}$$

cykloida: $x = a(t - \sin t)$

$y = a(1 - \cos t)$

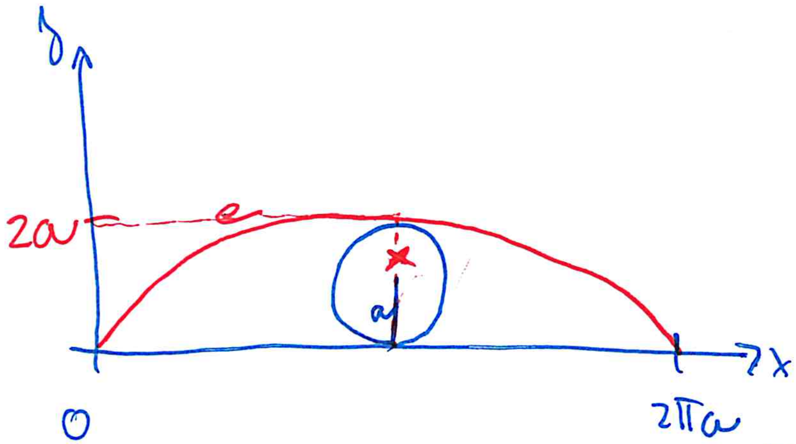
$t \in \langle 0, 2\pi \rangle$

$h(x,y) = 1$

$x_T = \pi a$

$m = h \cdot l = 8a$

$$y_T = \frac{1}{m}\int_l y \cdot h\, ds = \frac{1}{8a}\int_l y \cancel{h}\, ds$$



$x' = a(1 - \cos t)$

$y' = a \sin t$

$$ds = \sqrt{x'^2 + y'^2}\, dt = \sqrt{a^2(1 - 2\cos t + \cos^2 t) + a^2 \sin^2 t}\, dt =$$

$$= a\sqrt{2 - 2\cos t}\, dt$$

$$y_T = \frac{1}{8a}\int_{\ell} y\,ds = \frac{1}{8a}\int_0^{2\pi} a(1-\cos t)\,a\sqrt{2-2\cos t}\,dt =$$

$$y = a(1-\cos t)$$

$$t \in \langle 0, 2\pi \rangle$$

$$ds = a\sqrt{2-2\cos t}\,dt$$

$$= \frac{a\sqrt{2}}{8}\int_0^{2\pi}(1-\cos t)\cdot\sqrt{1-\cos t}\,dt =$$

$$= \frac{a\sqrt{2}}{8}\int_0^{2\pi}(1-\cos t)^{\frac{3}{2}}\,dt = \frac{a\cdot\sqrt{2}}{8}\int_0^{2\pi}(\underbrace{1-1}_{0}+2\sin^2\frac{t}{2})^{\frac{3}{2}}\,dt =$$

$$\cos t = \cos^2\frac{t}{2} - \sin^2\frac{t}{2} = 1 - 2\sin^2\frac{t}{2}$$

$$= \frac{a\cdot\sqrt{2}\cdot\sqrt{8}}{8}\int_0^{2\pi}(\sin^2\frac{t}{2})^{\frac{3}{2}}\,dt =$$

$$= \frac{a}{2}\int_0^{2\pi}\sin^3\frac{t}{2}\,dt = \frac{a}{2}\int_0^{2\pi}(1-\cos^2\frac{t}{2})\sin\frac{t}{2}\,dt = \frac{a}{2}\cdot\frac{8}{3} = \boxed{\frac{4}{3}a = y_T}$$

$$u = \cos\frac{t}{2}$$