1) $\vec{a} = (\underbrace{3x^2y - 3y^2}_{u(x,y)}, \underbrace{x^3 - 6xy}_{v(x,y)})$ je potenciálne v E_2 ?

$$\boxed{\frac{\partial u}{\partial y} \overset{?}{=} \frac{\partial v}{\partial x}}$$

$$\frac{\partial u}{\partial y} = 3x^2 - 6y$$

pole $\vec{a}$ je potenciálne

$$\frac{\partial v}{\partial x} = 3x^2 - 6y$$

2) $\vec{b} = \left(3x^2y - z^2 + 2z,\ x^3 + 2yz - 3,\ y^2 - 2xz + 2x\right)$

pole $\vec{b}$ je potenciálne $\longleftrightarrow$ $\operatorname{rot} \vec{b} = \vec{o}$

$\nabla = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)$ $\qquad$ $\nabla f = \operatorname{grad} f = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}\right)$

$$\cancel{\nabla \times}\ \operatorname{rot} \vec{b} = \vec{\nabla} \times \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 3x^2y - z^2 + 2z & x^3 + 2yz - 3 & y^2 - 2xz + 2x \end{vmatrix} =$$

$$\begin{matrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \end{matrix}$$

$$\boxed{\vec{b} \text{ je potenciálne}}$$

$$= \vec{i}\, 2y + \vec{k}\, 3x^2 + \vec{j}(-2z+2) - \vec{k}\, 3x^2 - \vec{i}\, 2y - \vec{j}(-2z+2) = \vec{o}$$

3) Spočítejte $\int_a x\,dx + y^2\,dy + xz\,dz = \circledast$, a je úsečka z $[0,1,0] = A$ do $[4,3,1] = B$

integrujeme pole $\vec{b} = (x, y^2, xz)$, $d\vec{r} = (dx, dy, dz)$ $\Big\}$ $\int_a \vec{b} \cdot d\vec{r}$

$$\underline{X = A + (B-A)t}$$
$$t \in \langle 0,1 \rangle$$

parametrické rovnice úsečky:

$$\boxed{\begin{array}{ll} x = 0 + 4t & dx = 4\,dt \\ y = 1 + 2t & dy = 2\,dt \\ z = 0 + 1t & dz = 1\,dt \end{array}}$$

$t \in \langle 0,1 \rangle$

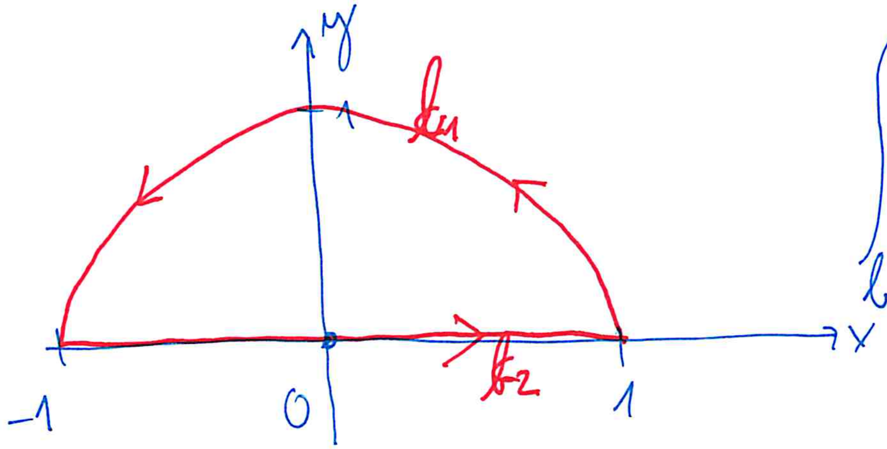
$$\circledast = \int_0^1 \Big(\underbrace{4t}_{x} \cdot \underbrace{4}_{dx} + \underbrace{(1+2t)^2}_{y^2} \underbrace{2}_{dy} + \underbrace{4t}_{x} \underbrace{t}_{z} \Big) dt =$$

$$= \int_0^1 \left(16t + 2 + 8t + 8t^2 + 4t^2 \right) dt =$$

$$= \int_0^1 \left(2 + 24t + 12t^2 \right) dt = \left[2t + 12t^2 + 4t^3 \right]_0^1 = 2 + 12 + 4 = \underline{\underline{18}}$$

4) $\int_{b} xy\,dx + 3x\,dy = \int_{b_1} xy\,dx + 3x\,dy + \int_{b_2} xy\,dx + 3x\,dy = \frac{3}{2}\pi$

$\frac{3}{2}\pi \qquad 0$



$$\int_{b_1} xy\,dx + 3x\,dy = \int_0^\pi (-\cos t \sin^2 t + 3\cos t \cos t)\,dt =$$

$$= \int_0^\pi (3\cos^2 t - \cos t \sin^2 t)\,dt = \frac{3}{2}\pi$$

$b_1)$: $x = \cos t \qquad dx = -\sin t\,dt$

$y = \sin t \qquad dy = \cos t\,dt$

$t \in \langle 0, \pi \rangle \qquad \cos 2t = \cos^2 t - \sin^2 t = 2\cos^2 t - 1$

$\cos^2 t = \frac{1 + \cos 2t}{2}$

$$3\int_0^{\pi} \cos^2 t\, dt = \frac{3}{2}\int_0^{\pi}(1+\cos 2t)\, dt = \frac{3}{2}\left(t + \sin 2t \cdot \frac{1}{2}\right)\Big|_0^{\pi} = \underline{\underline{\frac{3}{2}\pi}}$$

$$\int_0^{\pi} \sin^2 t \cos t\, dt = \int_0^0 u^2\, du = \underline{\underline{0}}$$

$u = \sin t$

$du = \cos t\, dt$

$b_2: \ [-1,0] \longrightarrow [1,0]$

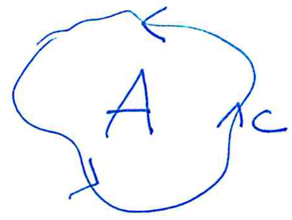
$$\int_{b_2} xy\, dx + 3x\, dy = \int_0^1 (\underbrace{-1+2t}_{x})\underbrace{0}_{y} + 3(\underbrace{-1+2t}_{x})0\, dt = 0$$

$x = -1 + 2t \qquad dx = 2\,dt$

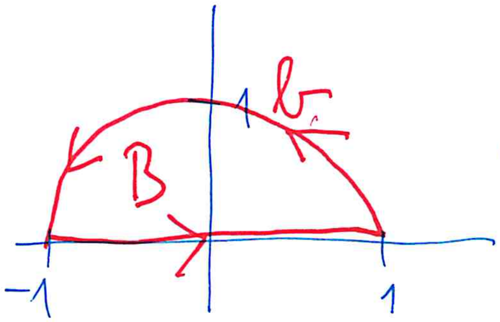
$y = 0 \qquad dy = 0\,dt$

$t \in \langle 0, 1 \rangle$

greenova věta: $\oint_c u\,dx + v\,dy = \iint_A \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) dx\,dy$



$$\int_b xy\,dx + 3x\,dy = \iint_B (3-x)\,dx\,dy = \iint_B (3-\rho\cos\varphi)\,\rho\,d\rho\,d\varphi =$$



$x = \rho\cos\varphi$

$y = \rho\sin\varphi$

$0 \le \rho \le 1$

$0 \le \varphi \le \pi$

$$= \int_0^\pi \left[\int_0^1 (3\rho - \rho^2\cos\varphi)\,d\rho\right] d\varphi =$$

$$= \int_0^\pi \left[3\frac{\rho^2}{2} - \frac{\rho^3}{3}\cos\varphi\right]_0^1 d\varphi = \int_0^\pi \left(\frac{3}{2} - \frac{1}{3}\cos\varphi\right) d\varphi =$$

$$= \left[\frac{3}{2}\varphi - \frac{1}{3}\sin\varphi\right]_0^\pi = \underline{\underline{\frac{3}{2}\pi}}$$

$$\boxed{S = \frac{1}{2}\oint_c (-y)\,dx + x\,dy} = \frac{1}{2}\int_0^{2\pi}\left(+a^2\sin^3 t\, 3\cos^2 t\,\sin t + 3a^2\cos^3 t\,\sin^2 t\cos t\right)dt =$$

G.V.

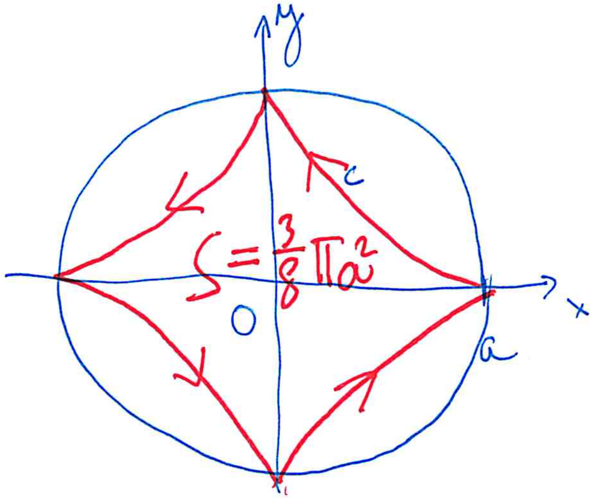
$$\frac{1}{2}\iint_A (1+1)\,dx\,dy = \iint_A 1\,dx\,dy = S$$

$x = a\cos^3 t \qquad dx = -a\,3\cos^2 t\,\sin t\,dt$

$y = a\sin^3 t \qquad dy = a\,3\sin^2 t\,\cos t\,dt$

$t \in \langle 0, 2\pi \rangle$

$$= \frac{3}{2}a^2\int_0^{2\pi}\left(\sin^4 t\cos^2 t + \cos^4 t\sin^2 t\right)dt =$$



$$= \frac{3}{2}a^2\int_0^{2\pi}\sin^2 t\cos^2 t\,dt = \frac{3}{8}a^2\int_0^{2\pi}\left(\sin^2 2t\right)dt =$$

$$\left(\frac{\sin 2t}{2}\right)^2 = \sin^2 t\cos^2 t$$

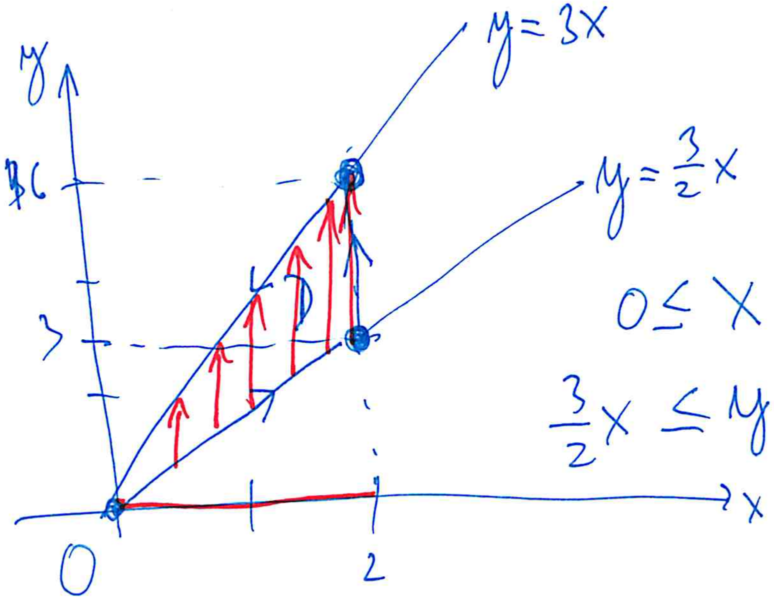
$$= \frac{3}{16}a^2\int_0^{2\pi}\left(1-\cos 4t\right)dt = \frac{3}{16}a^2\left[t - \frac{1}{4}\sin 4t\right]_0^{2\pi} = \boxed{\frac{3}{8}\pi a^2}$$

$$\cos 2t = \cos^2 t - \sin^2 t = 1 - 2\sin^2 t \rightarrow \sin^2 t = \frac{1-\cos 2t}{2}$$

$$\int_d xy^2\,dx + x^2\,dy = \iint_D (2x - 2xy)\,dx\,dy = \int_0^2 \left[\int_{\frac{3}{2}x}^{3x} (2x-2xy)\,dy\right] dx =$$

$F = (xy^2, x^2)$

d: kladně orientovaný obvod $\triangle$ $[0,0]$, $[2,3]$, $[2,6]$



$0 \le x \le 2$

$\frac{3}{2}x \le y \le 3x$

$$= \int_0^2 \left(3x^2 - \frac{27}{4}x^3\right) dx = \left[x^3 - \frac{27}{16}x^4\right]_0^2 =$$

$$= 8 - 27 = \underline{\underline{-19}}$$

$$\int_{\frac{3}{2}x}^{3x} (2x - 2xy)\,dy = \left[2xy - xy^2\right]_{\frac{3}{2}x}^{3x} = 6x^2 - 9x^3 - \left(3x^2 - \frac{9}{4}x^3\right) = 3x^2 - \frac{27}{4}x^3$$