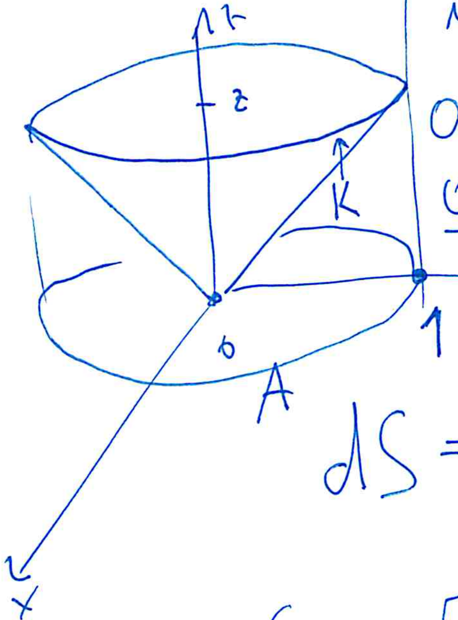
Moment setrvačnosti vzhledem k z plochy $z=\sqrt{x^2+y^2}$, $z=1$

$h(x,y,t)=1$



$x=\rho\cos\varphi$
$y=\rho\sin\varphi$
$0\le\rho\le 1$
$0\le\varphi\le 2\pi$

$$I_z=\iint_K (x^2+y^2)\,dS=\iint_A (x^2+y^2)\sqrt{2}\,dx\,dy = *$$

$$dS=\sqrt{1+\left(\frac{\partial f}{\partial x}\right)^2+\left(\frac{\partial f}{\partial y}\right)^2}\,dx\,dy=\sqrt{1+\frac{x^2}{x^2+y^2}+\frac{y^2}{x^2+y^2}}\,dx\,dy=\sqrt{2}\,dx\,dy$$

$$z=f(x,y)=\sqrt{x^2+y^2}$$

$$\frac{\partial f}{\partial x}=\frac{1}{\cancel{2}}\frac{\cancel{2}x}{\sqrt{x^2+y^2}}$$

$$\frac{\partial f}{\partial y}=\frac{y}{\sqrt{x^2+y^2}}$$

$$* = \iint_A \sqrt{2}\,\rho^2\cdot\rho\,d\rho\,d\varphi=\sqrt{2}\int_0^{2\pi}\left(\int_0^1\rho^3\,d\rho\right)d\varphi=$$

$$=\sqrt{2}\int_0^{2\pi}\left[\frac{\rho^4}{4}\right]_0^1 d\varphi=\frac{\sqrt{2}}{4}\left[\varphi\right]_0^{2\pi}=\boxed{\frac{\sqrt{2}}{2}\pi=I_z}$$

Stokesova věta: $\displaystyle\oint_C \vec{a}\cdot d\vec{r} = \iint_S \operatorname{rot}\vec{a}\cdot\vec{n}\, dS$

1) $\vec{a} = (-y, z, x)$ $\quad$ C: $x^2+y^2+z^2=R^2$ a $2x-y+2z=0$, $\quad \vec{n}=(2,-1,2)$

$\|\vec{n}\| = \sqrt{2^2+(-1)^2+2^2} = 3$

$\vec{n}_0 = \frac{1}{3}(2,-1,2)$

$$\oint_C \vec{a}\cdot d\vec{r} = \iint_S -(1,1,-1)\cdot\frac{1}{3}(2,-1,2)\,dS = \frac{1}{3}\iint_S -(2-1-2)\,dS =$$

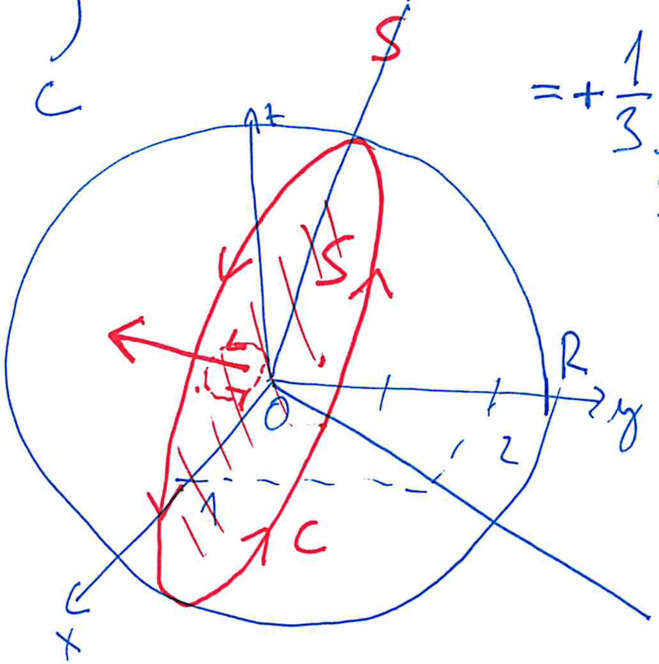
$$= +\frac{1}{3}\iint_S 1\,dS = \underline{\underline{\frac{1}{3}\pi R^2}}$$

obsah plochy S

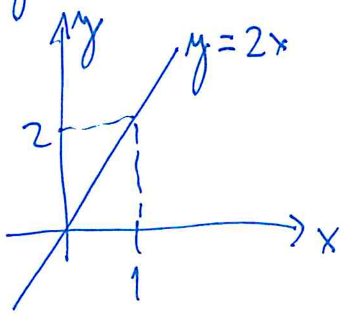
$$\operatorname{rot}\vec{a} = \begin{array}{|ccc|} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ -y & z & x \end{array} \begin{array}{c} \\ \end{array}$$

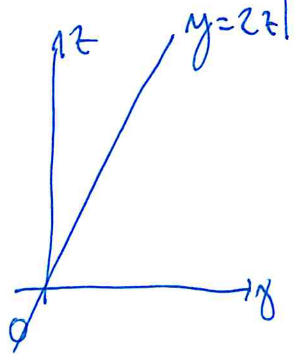
$$\begin{array}{ccc} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \end{array}$$

$= 0 - (-\vec{k}+\vec{i}+\vec{j}) = (1,1,-1)$



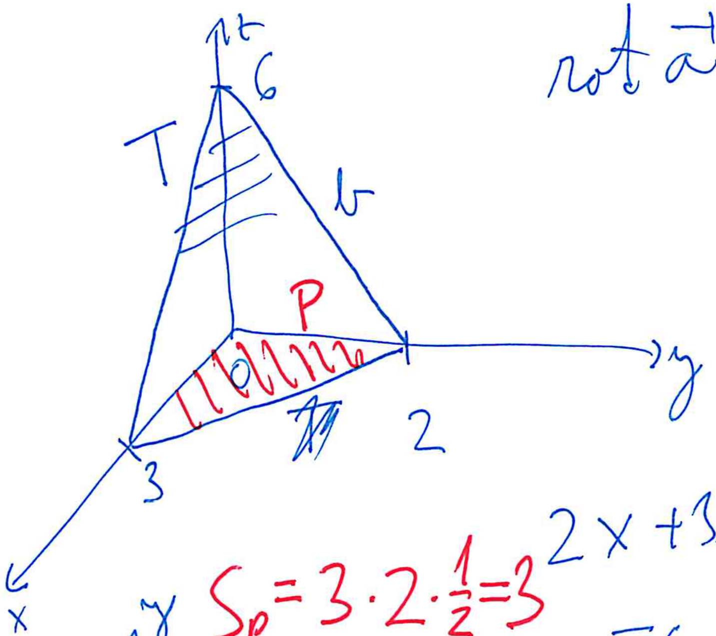
$y = 2x + 2z$





$\vec{a} = (x+1, x+y, 1-2z)$, b je Δ $[3,0,0]$, $[0,2,0]$, $[0,0,6]$, $\vec{n} = (2,3,1)$

$$\int_b \vec{a}\cdot d\vec{r} = \iint_T \operatorname{rot}\vec{a}\cdot\vec{n}\,dS = \iint_P (0,0,1)\cdot(2,3,1)\,dx\,dy = \iint_P 1\,dx\,dy = S_{\Delta P} = \underline{\underline{3}}$$

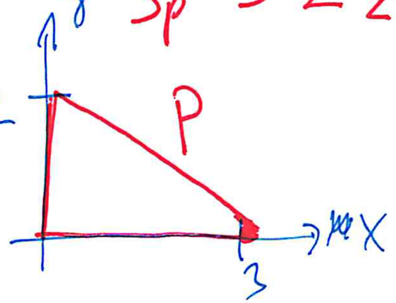


$$\operatorname{rot}\vec{a} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x+1 & x+y & 1-2z \end{vmatrix} = \vec{k} - (\vec{o}) = \vec{k} = (0,0,1)$$

$$\begin{matrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \end{matrix}$$

$2x + 3y + z = 6$

$S_P = 3\cdot 2\cdot\frac{1}{2} = 3$



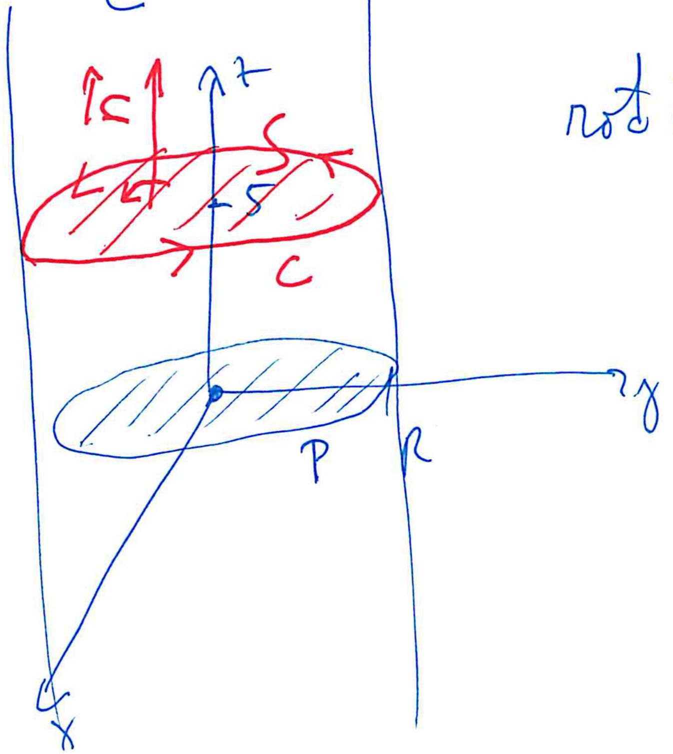
$F(x,y,z) = 2x + 3y + z - 6 = 0$

$\operatorname{grad} F = (2,3,1)$

$\vec{a} = (-y, x, z)$ $\quad C: x^2+y^2=R^2, \ \vec{n} = (0,0,1),$ $\quad \big| \ z=5: \ F(x,y,z) = z-5 = 0$

$\mathrm{Grad}\,F = (0,0,1)$

$$\int_C \vec{a}\cdot d\vec{r} = \iint_S \mathrm{rot}\,\vec{a}\cdot\vec{n}\,dS = \iint_P (0,0,2)\cdot(0,0,1)\,dx\,dy = \iint_P 2\,dx\,dy = 2S_P = \underline{\underline{2\pi R^2}}$$



$$\mathrm{rot}\,\vec{a} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ -y & x & z \end{vmatrix} \begin{matrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \end{matrix} = \vec{k} - (-\vec{k}) = 2\vec{k} = (0,0,2)$$

FG

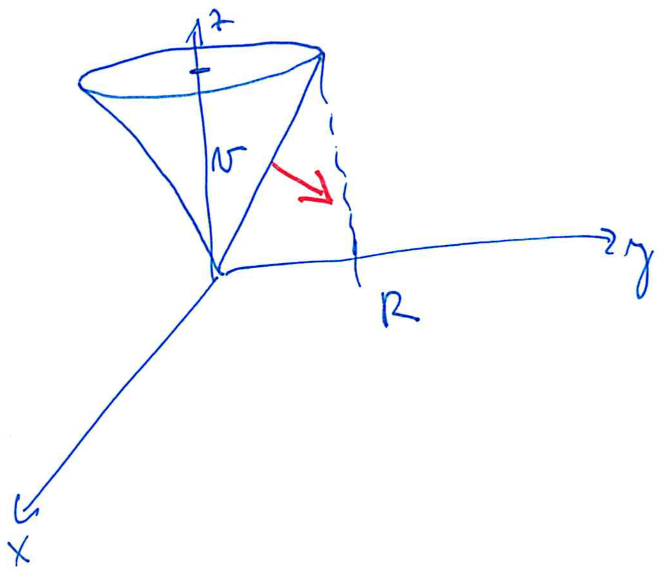
Gaussova věta: $$\iint\limits_S \vec{a}\cdot\vec{n}\, dS = \iiint\limits_A \operatorname{div}\vec{a}\; dx\,dy\,dz$$

S uzavřená plocha ohraničuje těleso A

$$\operatorname{div}\vec{a} = \vec{\nabla}\cdot\vec{a} = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)\cdot(u, v, w) = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}$$

tok $\vec{a} = (x, y, z)$ povrchem kužele s výškou v a poloměrem R

$$\operatorname{div}\vec{a} = \frac{\partial x}{\partial x} + \frac{\partial y}{\partial y} + \frac{\partial z}{\partial z} = 1 + 1 + 1 = 3$$

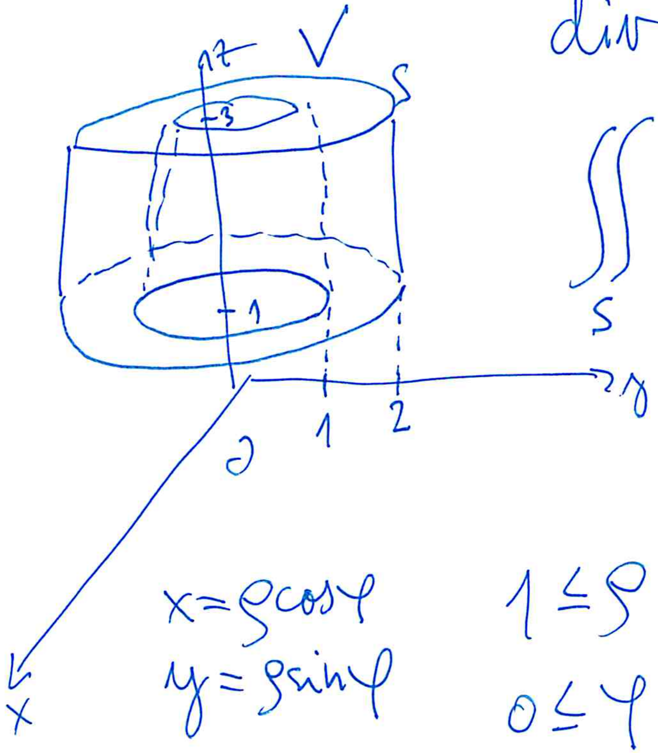


$$\iint\limits_S \vec{a}\cdot\vec{n}\, dS = \iiint\limits_A \operatorname{div}\vec{a}\; dx\,dy\,dz = \iiint\limits_A 3\, dx\,dy\,dz =$$

$$= 3V = \underline{\underline{\pi R^2 v}}$$

$\vec{a} = (xy^2,\ yz,\ x^2 z)$, $S:\ x^2+y^2=1,\ x^2+y^2=4,\ z=1,\ z=3$

$$\operatorname{div}\vec{a} = \frac{\partial xy^2}{\partial x} + \frac{\partial yz}{\partial y} + \frac{\partial x^2 z}{\partial z} = y^2 + z + x^2$$



$$\iint_S \vec{a}\cdot\vec{n}\,dS = \iiint_V (y^2+x^2+z)\,dx\,dy\,dz = \iiint_V (\rho^2+z)\rho\,d\rho\,d\varphi\,dz =$$

$$= \int_0^{2\pi}\left[\int_1^3\left(\int_1^2(\rho^3+\rho z)\,d\rho\right)dz\right]d\varphi =$$

$$= \int_0^{2\pi}\left(\int_1^3\left[\frac{\rho^4}{4}+\frac{\rho^2}{2}z\right]_1^2 dz\right)d\varphi = \int_0^{2\pi}\left(\int_1^3\left(\frac{15}{4}+\frac{3}{2}z\right)dz\right)d\varphi =$$

$$= \int_0^{2\pi}\left[\frac{15}{4}z+\frac{3}{4}z^2\right]_1^3 d\varphi = \int_0^{2\pi}\left(\frac{45}{4}-\frac{15}{4}+\frac{27}{4}-\frac{3}{4}\right)d\varphi = \frac{54}{4}\left[\varphi\right]_0^{2\pi} =$$

$$= \underline{\underline{27\pi}}$$

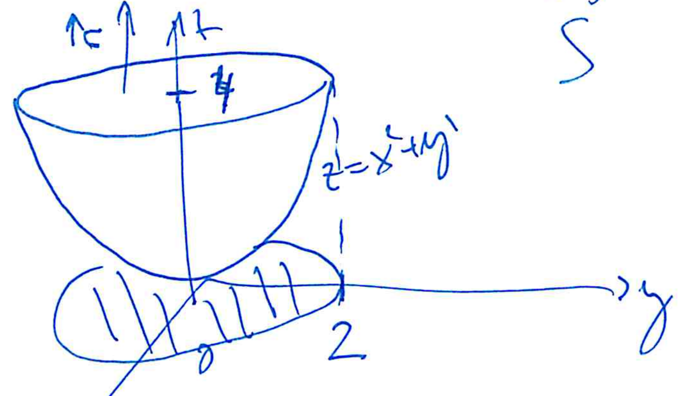
$x = \rho\cos\varphi$ $\quad 1 \le \rho \le 2$

$y = \rho\sin\varphi$ $\quad 0 \le \varphi \le 2\pi$

$z = z$ $\quad 1 \le z \le 3$

$\vec{a} = (x^3, z, y)$, S je povrch telesa A ohraničeného $z = x^2+y^2$, $z = 4$

$$\iint\limits_S \vec{a}\cdot\vec{n}\, dS = \iiint\limits_A \operatorname{div}\vec{a}\, dx\,dy\,dz = \iiint\limits_A 3x^2\, dx\,dy\,dz =$$



$$= \iiint\limits_A 3\rho^3\cos^2\varphi\, d\rho\, d\varphi\, dz =$$

$$= \int_0^{2\pi}\left[\int_0^2\left(\int_{\rho^2}^4 3\rho^3\cos^2\varphi\, dz\right)d\rho\right]d\varphi =$$

$$\operatorname{div}\vec{a} = \frac{\partial x^3}{\partial x} + \frac{\partial z}{\partial y} + \frac{\partial y}{\partial z} = \underline{3x^2}$$

$$= \int_0^{2\pi} 8(1+\cos 2\varphi)\, d\varphi = 8\left[\varphi - \frac{1}{2}\sin 2\varphi\right]_0^{2\pi}$$

$0 \le \rho \le 2$

$0 \le \varphi \le 2\pi$

$\rho^2 \le z \le 4$

$$= \underline{8\cdot 2\pi} = \underline{\underline{16\pi}}$$

1) $\int_{\rho^2}^{4} 3\rho^3\cos^2\varphi \, dz = 3\rho^3\cos^2\varphi \left[z\right]_{\rho^2}^{4} = 3\rho^3\cos^2\varphi\,(4-\rho^2) = (12\rho^3 - 3\rho^5)\cos^2\varphi$

2) $\int_{0}^{2} (12\rho^3 - 3\rho^5)\cos^2\varphi \, d\rho = \cos^2\varphi \left[3\rho^4 - \frac{\rho^6}{2}\right]_0^2 = \cos^2\varphi \cdot (48 - 32) =$

$= 16\cos^2\varphi = 8(1+\cos 2\varphi)$

$\cos 2\varphi = \cos^2\varphi - \sin^2\varphi = 2\cos^2\varphi - 1$

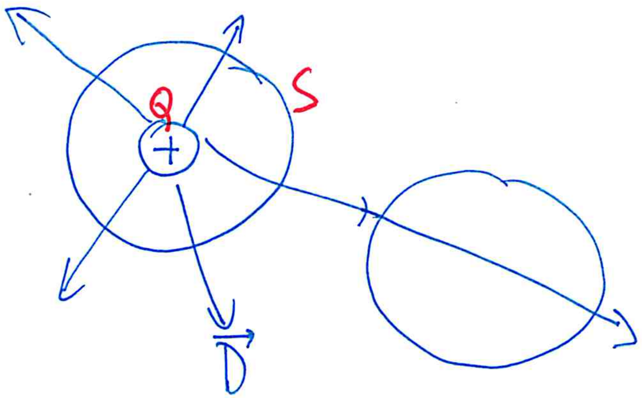
$\cos^2\varphi \doteq \frac{1+\cos 2\varphi}{2}$

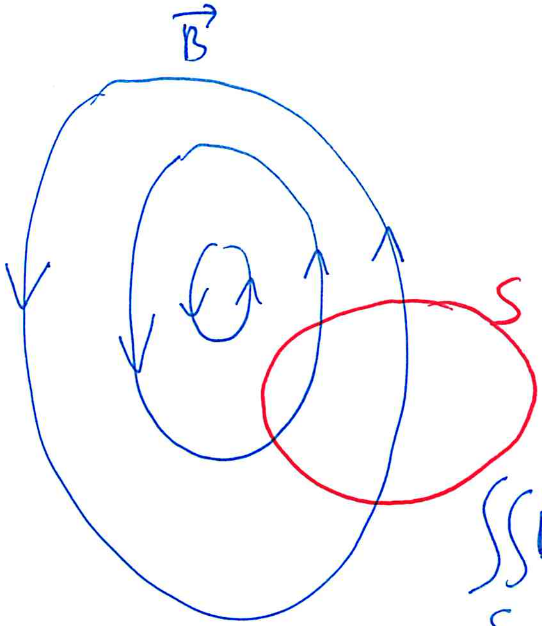
$$\operatorname{div} \vec{D} = \rho$$

$$\iiint_A \operatorname{div} \vec{D} \, dx\,dy\,dz = \iiint_A \rho \, dx\,dy\,dz$$

$\Big\downarrow$ 6. věta

$$\iint_S \vec{D} \cdot \vec{n} \, dS = Q$$





$$\iint_S \vec{B} \cdot \vec{n} \, dS = 0$$