Nalezněte Laplaceovy obrazy:

$$f_1(t) = 3t^4 - 5t^3 + 8$$

$$\mathcal{L}(f_1(t)) = \mathcal{L}(3t^4 - 5t^3 + 8) = 3\mathcal{L}(t^4) - 5\mathcal{L}(t^3) + 8\mathcal{L}(1) = (*)$$

$$\mathcal{L}(1) \to \frac{1}{s}$$

$$\mathcal{L}\left(\frac{t^4}{4!}\right) = \frac{1}{s^5} \to \frac{1}{4!}\mathcal{L}(t^4) = \frac{1}{s^5} \to \mathcal{L}(t^4) = \frac{4!}{s^5}$$

$$\mathcal{L}\left(\frac{t^3}{3!}\right) = \frac{1}{s^4} \longrightarrow \mathcal{L}(t^3) = \frac{3!}{s^4}$$

$$(*) = 3\frac{4!}{s^5} - 5\frac{3!}{s^4} + 8\frac{1}{s}$$

$$f_2(t) = e^{5t}$$

$$\mathcal{L}\left(e^{5t}\right) = \frac{1}{s-5}$$

$$f_3(t) = e^{-2t}\cos 5t$$

$$\mathcal{L}\left(e^{-2t}\cos 5t\right) = \frac{s+2}{(s+2)^2+25}$$

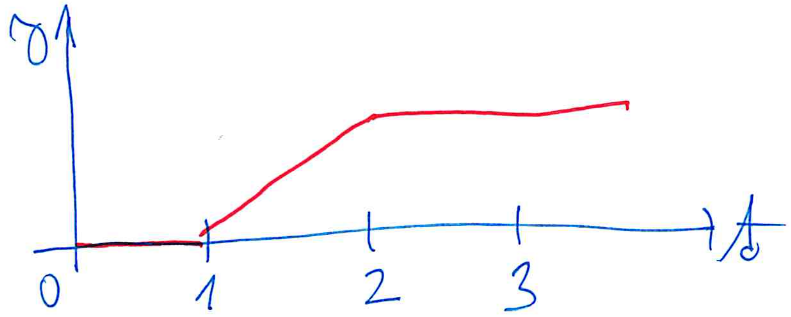
$$f_4(t) = t\sin 5t$$

$$\mathcal{L}\left(t\sin 5t\right) = \frac{10s}{(s^2+25)^2}$$

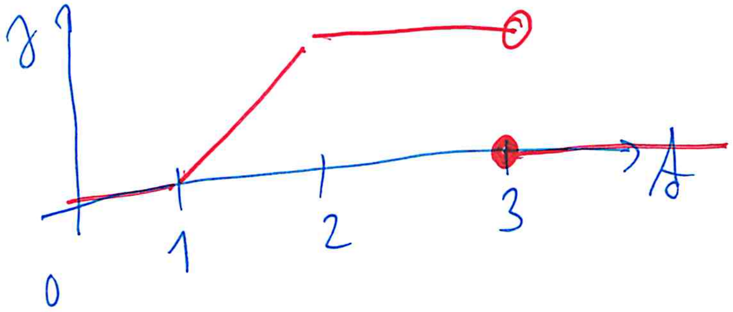
$$f_5(t) = t^7 e^{-3t}$$

$$\mathcal{L}\left(t^7 e^{-3t}\right) = \frac{7!}{(s+3)^8}$$

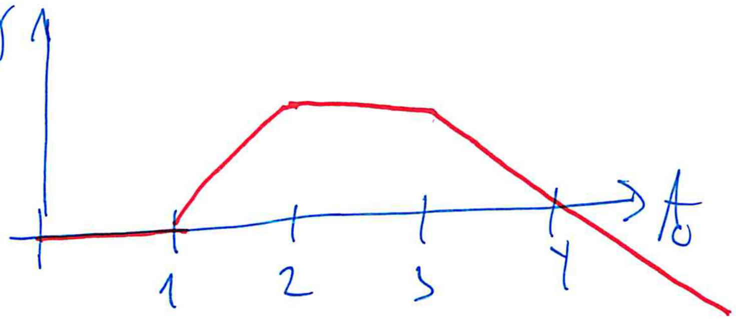
$$(t-1)u(t-1)-(t-1)u(t-2)+u(t-2)$$

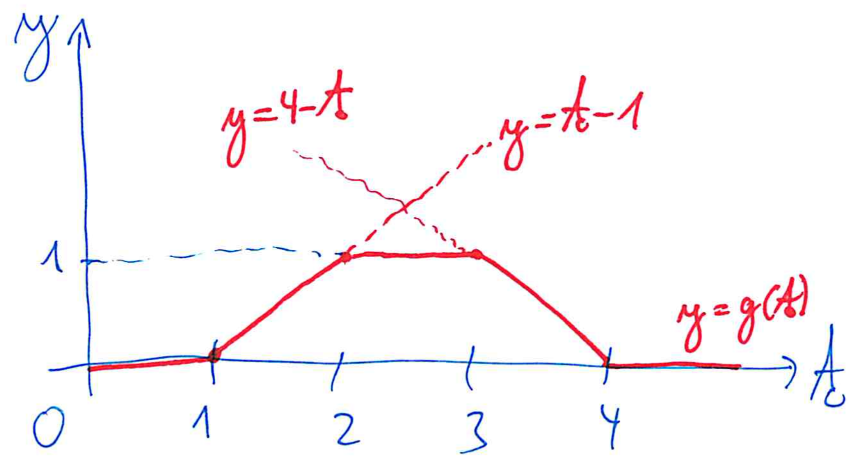


$$(t-1)u(t-1)-(t-1)u(t-2)+u(t-2)-u(t-3)$$



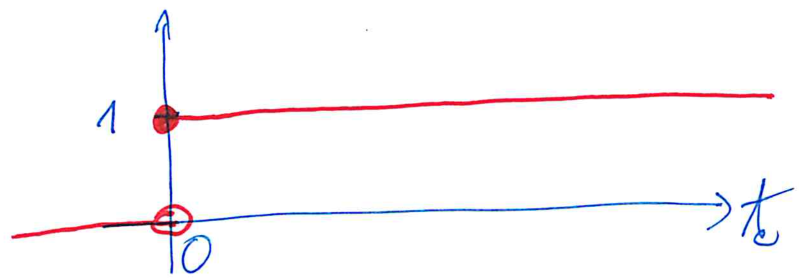
$$(t-1)u(t-1)-(t-1)u(t-2)+u(t-2)-u(t-3)+(4-t)u(t-3)$$



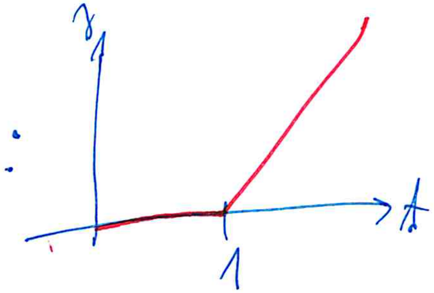


$u(t)=0 \quad t<0$

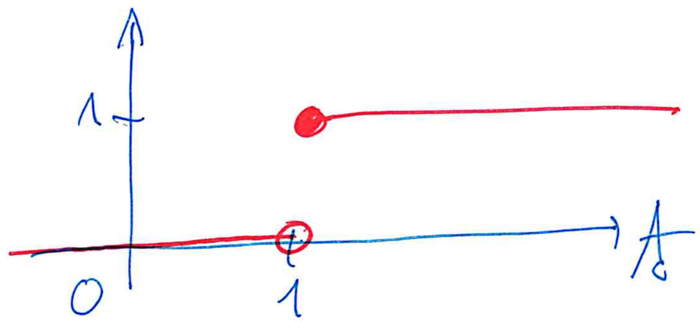
$u(t)=1 \ , \ t \geq 0$



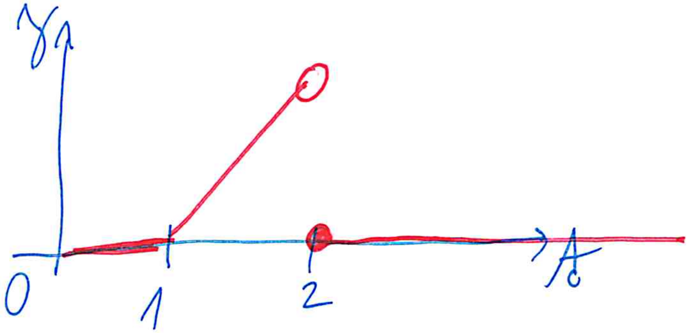
$(t-1)\,u(t-1)$:



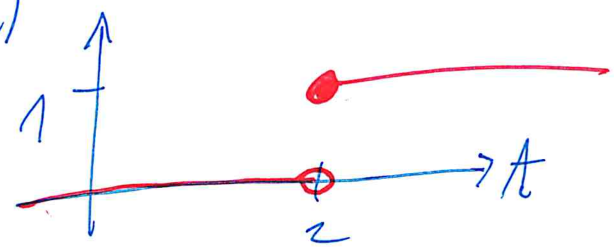
$y=u(t-1)$:



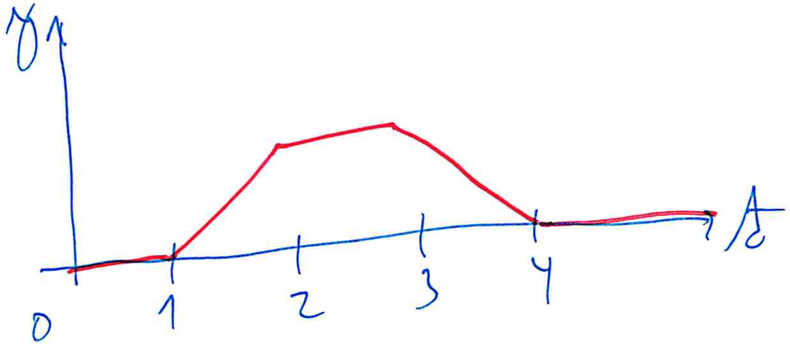
$(t-1)\,u(t-1)-(t-1)\,u(t-2)$



$y=u(t-2)$



$$(t-1)u(t-1)-(t-1)u(t-2)+u(t-2)-u(t-3)+(4-t)u(t-3)-(4-t)u(t-4)$$



$$g(t)=(t-1)u(t-1)-(t-2)u(t-2)-(t-3)u(t-3)+(t-4)u(t-4)$$

$$\mathcal{L}(g(t))=\frac{1}{s^2}e^{-s}-\frac{1}{s^2}e^{-2s}-\frac{1}{s^2}e^{-3s}+\frac{1}{s^2}e^{-4s}$$

$$F_1(s) = \frac{1}{(s+1)^3}$$ zpětná transformace

$$\mathcal{L}^{-1}\left(\frac{1}{(s+1)^3}\right) = \frac{t^2 e^{-t}}{2}$$

$$F_4(s) = \frac{1}{s^2+4s+5} = \frac{1}{(s+2)^2+1}$$

$$\mathcal{L}^{-1}\left(\frac{1}{(s+2)^2+1}\right) = e^{-2t} \sin \cancel{1} t$$

$$F_3(s) = \frac{s}{(s-2)(s-3)} = \frac{A}{s-2} + \frac{B}{s-3} = \frac{\frac{2}{5}}{s-2} + \frac{\frac{3}{5}}{s-3}$$

$$\mathcal{L}^{-1}\left(\frac{s}{(s-2)(s-3)}\right) = \frac{2}{5}\mathcal{L}^{-1}\left(\frac{1}{s-2}\right) + \frac{3}{5}\mathcal{L}^{-1}\left(\frac{1}{s-3}\right)$$

$$= \frac{2}{5}e^{2t} + \frac{3}{5}e^{3t}$$

$$y'' + y' + 4y = \cos 2t, \quad y(0) = 0, \quad y'(0) = 1$$

$$\mathcal{L}(\cos 2t) = \frac{s}{s^2+4}$$

$$\mathcal{L}(y) = Y$$

$$\mathcal{L}(y') = sY - 0$$

$$\mathcal{L}(y'') = s^2 Y - s \cdot 0 - 1$$

obraz rovnice:

$$\underbrace{s^2 Y - 1}_{y''} + \underbrace{sY}_{y'} + \underbrace{4Y}_{4y} = \underbrace{\frac{s}{s^2+4}}_{\cos 2t} \Big/ +1$$

$$Y(s^2+s+4) = \frac{s}{s^2+4} + 1$$

$$Y = \frac{s}{(s^2+4)(s^2+s+4)} + \frac{1}{s^2+s+4} =$$

$$= \frac{\cancel{s+s^2+4}}{(s^2+4)\cancel{(s^2+s+4)}} = \frac{1}{s^2+4}$$

$$\boxed{y = \frac{1}{2}\sin 2t}$$