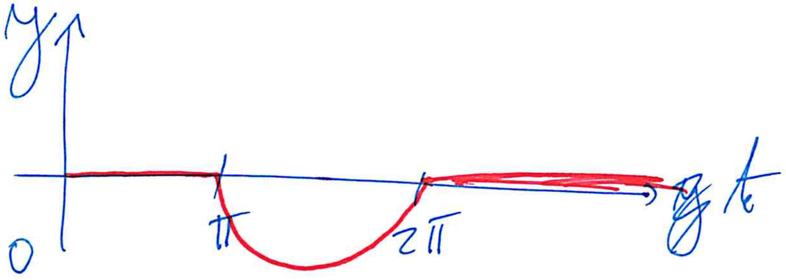
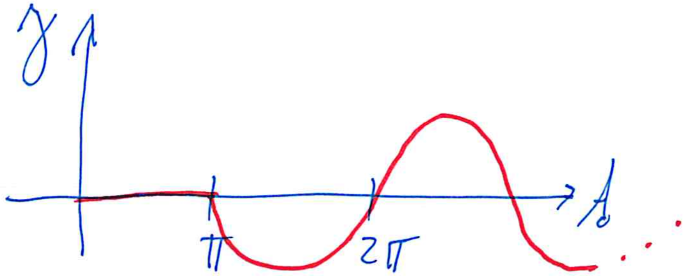
$$y'' + 4y = f(t), \quad y(0) = 1, \quad y'(0) = 4$$

$$f(t) = \sin t \quad t \in \langle \pi, 2\pi)$$

$$f(t) = 0 \quad \cancel{t} \text{ jinak}$$



$\sin t \cdot u(t-\pi)$



$\sin t = -\sin(t-\pi)$

$\sin t = \sin(t-2\pi)$

$$f(t) = \sin t \cdot u(t-\pi) - \sin t \, u(t-2\pi)$$

$$f(t) = -\sin(t-\pi) \cdot u(t-\pi) - \sin(t-2\pi) \, u(t-2\pi)$$

$$y'' + 4y = \overbrace{-\sin(t-\pi)\,u(t-\pi) - \sin(t-2\pi)\,u(t-2\pi)}^{f(t)}, \quad y(0)=1, \; y'(0)=4$$

$$\mathcal{L}(y) = Y, \quad \mathcal{L}(y'') = s^2 Y - s - 4$$

$$\mathcal{L}(f(t)) = -\frac{1}{s^2+1}e^{-\pi s} - \frac{1}{s^2+1}e^{-2\pi s}$$

dosaz. rovnice: $s^2 Y - s - 4 + 4Y = -\frac{1}{s^2+1}e^{-\pi s} - \frac{1}{s^2+1}e^{-2\pi s}$

$$Y(s^2+4) = -\frac{1}{s^2+1}e^{-\pi s} - \frac{1}{s^2+1}e^{-2\pi s} + s + 4 \quad \Big/ :(s^2+4)$$

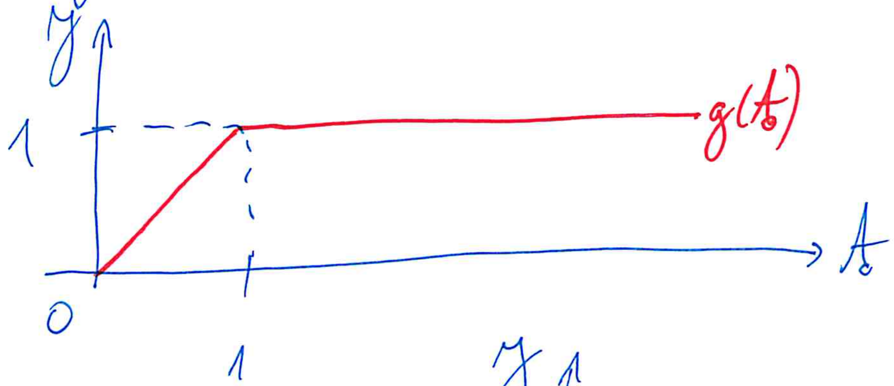
$$Y = -\frac{1}{\underbrace{(s^2+1)}_{a=1}\underbrace{(s^2+4)}_{b=2}}e^{-\pi s} - \frac{1}{(s^2+1)(s^2+4)}e^{-2\pi s} + \frac{s}{s^2+4} + \frac{4}{s^2+4}$$

$$y = -\frac{2\sin(t-\pi) - \sin 2(t-\pi)}{2\cdot 3}\,u(t-\pi) - \frac{2\sin(t-2\pi) - \sin 2(t-2\pi)}{2\cdot 3}\cdot u(t-2\pi) +$$

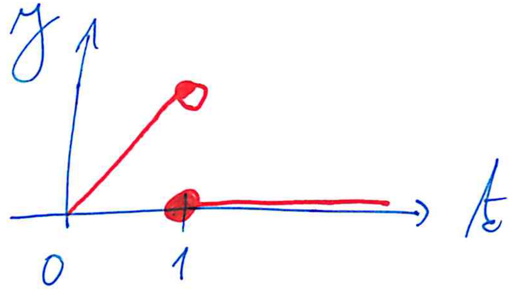
$$+ \cos 2t + 2\sin 2t$$

$y' - 2y = g(t)$, $y(0) = -1$ $\quad g(t) = t \quad t \in \langle 0, 1)$

$g(t) = 1 \quad t \geq 1$



$t - t\,u(t-1)$



$$g(t) = t - t\,u(t-1) + u(t-1) = t - u(t-1)(t-1)$$

$$\mathcal{L}(g(t)) = \frac{1}{s^2} - \frac{1}{s^2} e^{-s}$$

$$y' - 2y = t - u(t-1)(t-1) \quad , y(0) = -1$$

$$\mathcal{L}(g(t)) = \frac{1}{s^2} - \frac{1}{s^2} e^{-s}$$

$$\mathcal{L}(y) = Y \quad , \mathcal{L}(y') = sY + 1$$

obraz rovnice: $$sY + 1 - 2Y = \frac{1}{s^2} - \frac{1}{s^2} e^{-s}$$

$$Y(s-2) = \frac{1}{s^2} - \frac{1}{s^2} e^{-s} - 1$$

$$Y = \frac{1}{s^2(s-2)}^{*} - \frac{1}{s^2(s-2)} e^{-s} - \frac{1}{s-2}$$

$$y = \underbrace{-\frac{1}{2}t - \frac{1}{4} + \frac{1}{4}e^{2t}}_{*\ \text{(další str.)}} - \left(-\frac{1}{2}(t-1) - \frac{1}{4} + \frac{1}{4}e^{2(t-1)}\right)u(t-1) - e^{2t}$$

$$\frac{*\,1}{s^2(s-2)} = \frac{A}{s^2} + \frac{B}{s} + \frac{C}{s-2} = \frac{A(s-2) + B(s^2-2s) + Cs^2}{s^2(s-2)} =$$

$u\ s^2:\ B + C = 0 \qquad C = \frac{1}{4}$

$u\ s:\ A - 2B = 0 \rightarrow B = -\frac{1}{4}$

$u\ s^0:\ -2A = 1 \rightarrow \underline{\underline{A = -\frac{1}{2}}}$

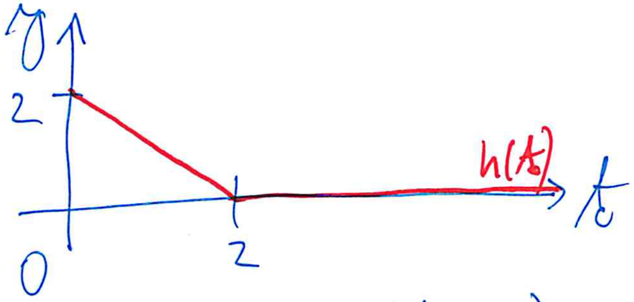
$$= \frac{-\frac{1}{2}}{s^2} + \frac{-\frac{1}{4}}{s} + \frac{\frac{1}{4}}{s-2}$$

$$-\frac{1}{2}t - \frac{1}{4} + \frac{1}{4}e^{2t} \quad *$$

$$y'' + 16y = h(t), \quad y(0) = 1, \; y'(0) = 0$$

$$h(t) = 2 - t \quad \text{pro} \quad t \in \langle 0, 2)$$

$$h(t) = 0 \quad \text{pro} \quad t \geq 2$$



$$h(t) = 2 - t - (2 - t)\,u(t-2)$$

$$h(t) = 2 - t + (t-2)\,u(t-2)$$

$$\mathcal{L}(h(t)) = \frac{2}{s} - \frac{1}{s^2} + \frac{1}{s^2} e^{-2s}$$

$$\mathcal{L}(y) = Y, \quad \mathcal{L}(y'') = s^2 Y - s$$

obraz równie: $s^2Y - s + 16Y = \frac{2}{s} - \frac{1}{s^2} + \frac{1}{s^2}e^{-2s} \quad / + s$

$$Y(s^2+16) = \frac{2}{s} - \frac{1}{s^2} + \frac{1}{s^2}e^{-2s} + s \quad / : (s^2+16)$$

$$Y = \frac{2}{s(s^2+16)} - \frac{1}{s^2(s^2+16)} + \frac{1}{s^2(s^2+16)}e^{-2s} + \frac{s}{s^2+16}$$

$$y = 2\,\frac{1-\cos 4t}{16} - \frac{4t - \sin 4t}{64} + \frac{4(t-2) - \sin 4(t-2)}{64}\,u(t-2) + \cos 4t$$

$$y_1' - y_2 = 1 \qquad y_1(0) = 1,\ y_2(0) = 0$$

$$y_2' + y_1 = t \qquad *)\ y_1 = t - y_2' = t + \cos t = y_1$$

$$\mathcal{L}(y_1) = Y_1 \qquad \mathcal{L}(y_1') = sY_1 - 1 \qquad \mathcal{L}(1) = \frac{1}{s}$$

$$\mathcal{L}(y_2) = Y_2 \qquad \mathcal{L}(y_2') = sY_2 \qquad \mathcal{L}(t) = \frac{1}{s^2}$$

obraz soustavy:

$$\left.\begin{aligned} sY_1 - 1 - Y_2 &= \frac{1}{s} \\ sY_2 + Y_1 &= \frac{1}{s^2} \quad /(-s) \end{aligned}\right\} \begin{aligned} &-Y_2 - s^2 Y_2 = \frac{1}{s} + 1 - \frac{s}{s^2} = 1 \\ &Y_2(-1 - s^2) = 1 \rightarrow Y_2 = \frac{-1}{s^2+1} \end{aligned}$$

$$y_2 = -\sin t \quad *$$

$$y_1' + y_2 = e^{-2t} \qquad y_1(0) = y_2(0) = 0$$

$$y_2' - y_1 + 2y_2 = e^{-2t}$$

$$\mathcal{L}(y_1) = Y_1 \quad \mathcal{L}(y_1') = sY_1 \quad \mathcal{L}(y_2) = Y_2, \ \mathcal{L}(y_2') = sY_2, \ \mathcal{L}(e^{-2t}) = \frac{1}{s+2}$$

obraz soustavy: $sY_1 + Y_2 = \frac{1}{s+2}$

$$\left.\begin{aligned} sY_1 + Y_2 &= \frac{1}{s+2} \\ sY_2 - Y_1 + 2Y_2 &= \frac{1}{s+2} \quad / \cdot s \end{aligned}\right\} \quad Y_2 + s^2 Y_2 + 2sY_2 = \frac{s+1}{s+2}$$

$$Y_2(s^2 + 2s + 1) = \frac{s+1}{s+2}$$

$$sY_1 = \frac{1}{s+2} - Y_2 = \frac{1}{s+2} - \frac{1}{(s+1)(s+2)}$$

$$Y_2(s+1)^2 = \frac{s+1}{s+2} \quad / : (s+1)^2$$

$$\cancel{s}Y_1 = \frac{\cancel{s}}{(s+1)(s+2)} = Y_2$$

$$Y_2 = \frac{1}{(s+1)(s+2)} \qquad y_2 = \frac{e^{-t} - e^{-2t}}{1}$$

$$y_1 = y_2 = e^{-t} - e^{-2t}$$